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*To Thomas Stewart Wesner*

*Dad*

*To Margot*

*Phil*

# Contents

C H A P T E R 5

## The Trigonometric Functions



C H A P T E R 6

## Radian Measure, Properties of the Trigonometric and Inverse Trigonometric Functions



C H A P T E R 7

## Trigonometric Equations



C H A P T E R 8

## Additional Topics in Trigonometry



C H A P T E R 9

## Exponential and Logarithmic Functions

## CHAPTER 10 Systems of Linear Equations and Inequalities



## CHAPTER 11 The Conic Sections



## CHAPTER 12 Topics in Discrete Mathematics


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C H A P T E R

# 2 Equations and Inequalities

## 2–1 Linear equations

A book of the sixth century A.D., called the *Greek Anthology,* contains the following problem: If one pipe fills a cistern in one day, a second in two days, a third in three days, and a fourth in four days, how long will it take all four running together to fill it?

This section introduces the techniques necessary to solve this ancient problem.

### Linear equations in one variable

An **equation** is a statement that two expressions are equal. Thus, $5(2) = 10$ and $5(2) = 12$ are both equations.[1] The first is true and the second is false. The equation $5x = 10$ is neither true nor false; such equations are called **conditional equations.** The left side of an equation is the **left member,** and the right side is the **right member.**

In this section we are concerned with **first-degree conditional equations in one variable,** also called **linear equations in one variable.**[2] In such an equation the exponent of the variable is one.

If the variable $x$ in the equation $5x = 10$ is replaced by a number, the result is an equation that is true or false. The equation is true only if $x$ is replaced by 2. Any replacement value of the variable, such as 2 here, for which the equation is true is called a **root** or **solution** of the equation. The set of all solutions to an equation is called the **solution set.** To solve the equation means to find the solution set.

[1]The = symbol was first used by Robert Recorde in *The Whetstone of Witte,* published in 1557. "I will sette as I doe often in woorke use, a paire of paralleles, or Gemowe [twin] lines of one lengthe, thus: ═══, bicause noe 2 thynges, can be moare equalle."

[2]The algebra of ancient Egypt (some 4,000 years ago) was much concerned with linear equations.

If the solution set to an equation is all permissible replacement values of the variable, the equation is an **identity.** The equation $x + x = 2x$ is an identity because the left member equals the right member regardless of the value that $x$ represents. In this case the solution set is $R$, the set of real numbers. The equation $\frac{2x}{x} = 2$ is true for any value of $x$ except zero, which is not permissible. Thus, this is an identity for all real numbers except zero.

We solve a linear equation by forming a sequence of equivalent equations, until we come to one that is sufficiently simple to solve by inspecting it. Two equations are said to be **equivalent equations** if they have the same solution set. We form these equations by using the following two properties of equality.

**Addition property of equality**

For any algebraic expressions $A$, $B$, and $C$,

$$\text{if } A = B \text{ then } A + C = B + C$$

**Multiplication property of equality**

For any algebraic expressions $A$, $B$, and $C$,

$$\text{if } A = B \text{ then } AC = BC$$

One additional property that is useful is called cross multiplication for equations.

**Cross multiplication for equations**

For any real numbers $a$, $b$, $c$, $d$, $b \neq 0$ and $d \neq 0$,

$$\text{if } \frac{a}{b} = \frac{c}{d} \text{ then } ad = bc$$

**Note** The proof of this property is left as an exercise. It can be visualized as if $\frac{a}{b} \times \frac{c}{d}$ then $ad = bc$

The cross-multiplication property states that the products formed across the two diagonals are equal. For example, if $\frac{x+1}{3} = \frac{5}{8}$, then we can conclude that $8(x + 1) = 15$. This property *only* applies when *one* fraction equals *one* other fraction. It does not apply to the equation $\frac{x+1}{3} + \frac{1}{3} = \frac{5}{8}$, for example, because the left side is not simply one fraction.

These properties allow us to perform the usual transformations on equations, as illustrated below. The basic procedure for solving linear equations follows.

**Solving linear equations**

- Clear any denominators by multiplying each term by the least common denominator of all the terms (or cross multiply if possible).
- Perform any indicated multiplications (remove parentheses).
- Use the addition property of equality so that all terms with the variable are in one member of the equation, and all other terms are in the other member.
- If necessary factor out the variable from the terms containing it.
- Divide both members of the equation by the coefficient of the variable.

Observe how these steps are used in examples 2-1 A, 2-1 B, and 2-1 C.

■ ***Example 2–1 A***

Find the solution set.

**1.** $\dfrac{x-2}{3} = \dfrac{x}{6}$

$$\begin{aligned} 6(x-2) &= 3x \\ 6x - 12 &= 3x \\ 6x - 3x &= 12 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

$\{4\}$

**2.** $\dfrac{5}{6}x - \dfrac{3}{4} = 11 - \dfrac{x}{2}$

$$12\left(\frac{5}{6}x - \frac{3}{4}\right) = 12\left(11 - \frac{x}{2}\right)$$
$$\begin{aligned} 10x - 9 &= 132 - 6x \\ 16x &= 141 \\ x &= \tfrac{141}{16} = 8\tfrac{13}{16} \end{aligned}$$

$\{8\frac{13}{16}\}$

**3.** $-2(x-4) + 1 = 3(x+3) - 5x$

$$\begin{aligned} -2x + 8 + 1 &= 3x + 9 - 5x \\ 0 &= 0 \end{aligned}$$

This is a true statement regardless of the value of $x$, indicating the solution set is $R$ and that the equation is an identity.

$R$ Solution set

**4.** $5x - 4(x-3) = x + 11$

$$\begin{aligned} x + 12 &= x + 11 \\ 12 &= 11 \end{aligned}$$

This statement is a contradiction—that is, it is never true, regardless of the value of $x$; thus, the solution set is the empty set, indicated by the symbol $\phi$.

$\phi$ Solution set ■

**Note** A set which contains no elements is called the **empty set,** $\phi$.

Parts 3 and 4 of example 2–1 A illustrate *how to recognize an identity and an empty solution set.* In the first case we arrive at a statement that is true independent of the value of $x$, such as $1 = 1$ or $2x = x + x$, and in the second case we arrive at a contradiction, such as $1 = 0$.

■ ***Example 2–1 B***

Find a four-digit approximation to the solution to the equation $3.5x - 4.1(2x - 3) = 7.04$.

$$3.5x - 8.2x + 12.3 = 7.04$$
$$-4.7x = -5.26$$
$$x = \frac{-5.26}{-4.7} \approx 1.119$$
$$\{1.119\}$$

■

A graphing calculator can be used to verify numeric solutions to linear equations. This is a special case of what is covered in the *Computer-Aided Mathematics* section. For example, part 2 of example 2–1 A can be verified by graphing the two equations $Y_1 = \frac{5}{6}x - \frac{3}{4}$ and $Y_2 = 11 - \frac{x}{2}$ (see figure 2–1). At the point where the two lines cross $Y_1 = Y_2$, so $\frac{5}{6}x - \frac{3}{4} = 11 - \frac{x}{2}$. Therefore, the $x$-value at this point is the solution to the equation. Use the trace and zoom features to move the cursor to the point at which the lines cross. This will show that these two graphs cross at $x \approx 8.74$, which is close to the exact solution $8\frac{13}{16}$.

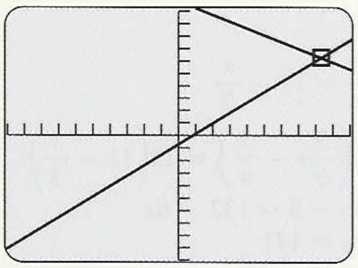


**Figure 2–1**

Thus, by graphing the right member of an equation and the left member we can find approximate numeric solutions using the trace calculator feature to find the value of $x$ where the two graphs cross. Note that if the lines cross somewhere off the graphing area the RANGE must be adjusted and the lines regraphed. This process can be tedious, and thus this graphing solution method is not too practical.

Unfortunately the graphing calculator cannot be used to verify the solutions to literal equations, which we now examine.

## Literal equations and formulas

A **literal equation** is one in which the solution is expressed in terms of nonnumeric symbols (letters). We may use the word **formula** for a literal equation in which the variables apply to some known situation.

To solve a literal equation for a variable means to rewrite it so it expresses that variable in terms of the others, by isolating that variable as the only term in one member of the equation. Literal equations that are linear in the unknown are solved *using the same steps* for solving linear equations *stated above.*

■ ***Example 2–1 C***

Solve the literal equation for the specified variable.

**1.** The formula for the perimeter of a rectangle is $P = 2\ell + 2w$. Solve for $\ell$.

$$P = 2\ell + 2w$$
$$P - 2w = 2\ell$$
$$\frac{P - 2w}{2} = \ell$$
$$\ell = \frac{P - 2w}{2}$$

**2.** $\dfrac{x + y}{x - y} = z$; solve for $x$.

$$\frac{x + y}{x - y} = z$$
$$(x - y)\left(\frac{x + y}{x - y}\right) = z(x - y)$$
$$x + y = zx - zy$$
$$x - zx = -zy - y$$
$$x(1 - z) = -y(z + 1)$$
$$\frac{x(1 - z)}{1 - z} = \frac{-y(z + 1)}{1 - z}$$
$$x = \frac{-y(z + 1)}{1 - z}$$

**Note** The expression for $x$ could also be written $\dfrac{y(z + 1)}{z - 1}$ by multiplying the numerator and denominator by $-1$. ■

## Interest problems

One class of problems for which linear equations are appropriate is **interest problems.** To solve these requires an understanding of the basic principles of simple interest. The formula $I = Prt$ describes the interest $I$, which is earned by a principal $P$ at a simple yearly interest rate $r$, in $t$ years.

If $t = 1$ (year), as it will be below, the formula is

$$I = Pr$$

For example, \$850 at 6% yields $I = (850)(0.06) = 51.00$. Thus, \$850 at 6% yearly interest rate produces \$51 interest.

Now suppose \$5,000 was invested in two accounts; \$1,000 at 3% and the remainder at 8%. The total interest would be

$$I = (1{,}000)(0.03) + (5{,}000 - 1{,}000)(0.08) = \$350.00$$

Now suppose that \$5,000 is invested, part at 3% and the rest at 8%, and that the interest earned is \$200. How much is invested at each rate? If $x$ is the amount invested at 3% (like the \$1,000 above) then $5{,}000 - x$ is the rest, invested at 8% (like the 5,000 − 1,000 above). Then the equation for $I = \$200$ is

$$200 = 0.03x + 0.08(5{,}000 - x)$$

which can be solved to find $x$, the amount invested at 3%:

$$200 = 0.03x + 400 - 0.08x \qquad \text{Multiply } 0.08(5{,}000 - x)$$
$$-200 = -0.05x \qquad \text{Combine terms, add } -400 \text{ to both members}$$
$$\frac{-200}{-0.05} = x \qquad \text{Divide each member by } -0.05$$
$$x = 4{,}000.$$

Thus, \$4,000 is invested at 3%, and \$1,000 (\$5,000 − \$4,000) at 8%.

■ ***Example 2–1 D***

A total of \$15,000 is invested, part at 5% and the remainder at 7.5%. If the total interest earned in a year is \$825, how much was invested at each rate?

If $x$ represents the amount invested at 5%, then $15{,}000 - x$ was invested at 7.5%. The amount of interest earned by the 5% investment is $0.05x$, and the amount of interest earned by the 7.5% investment is $0.075(15{,}000 - x)$. These two amounts of interest total \$825; this is expressed by the equation:

$$\text{Interest at 5\%} + \text{Interest at 7.5\%} = \text{Total interest}$$
$$\overbrace{0.05x}^{5\%} + \overbrace{0.075(15{,}000 - x)}^{7.5\%} = 825$$

15,000

$$0.05x + 1125 - 0.075x = 825$$
$$-0.025x = -300$$
$$x = \frac{-300}{-0.025} = 12{,}000$$

Thus, \$12,000 is invested at 5%, and the remainder, \$15,000 − \$12,000 = \$3,000, is invested at 7.5%. ■

## Mixture problems

A similar process is used to solve problems about mixtures. It is important to understand the physical setting for such problems. For example, what does it mean to say that a 20-liter container is full of an 8% acid/water solution? It means that 8% of the 20 liters is acid and the rest, 92%, is water. Thus, $(0.08)(20) = 1.6$ liters are acid, and $(0.92)(20) = 18.4$ liters are water. If we could separate the acid and water, we could represent the situation as shown in figure 2–2.

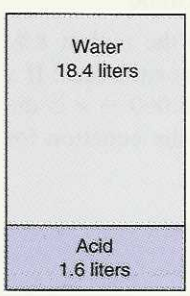


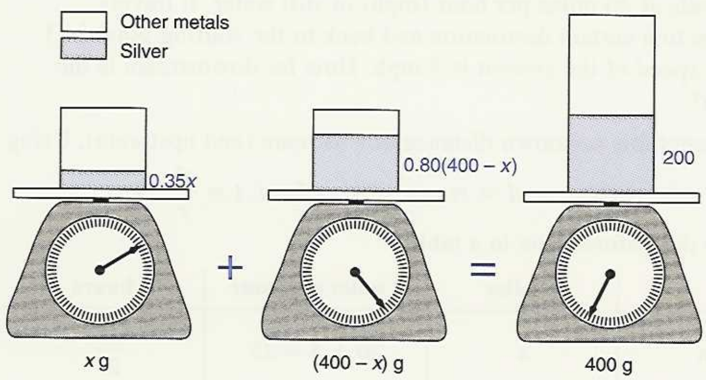

**Figure 2–2**

The following example illustrates a mixture problem. Just as with interest problems, we have information about several quantities that are percentages of other amounts. We usually obtain an equation by adding these percentages and equating them to some known total. Note that we do not add the *percents,* such as 80% + 35%. We do add *percentages,* such as 80% of one quantity + 35% of another quantity.

■ ***Example 2–1 E***

What quantities of an 80% silver alloy and a 35% silver alloy must be mixed to obtain 400 grams of a 50% silver alloy?

A problem like this can often be solved by writing an algebraic statement about the amount of silver present. The total amount of silver in the resulting mixture is 50% of the 400 grams, or $0.50(400) = 200$ grams. Where does this 200 grams of silver come from? It comes from the 80% and 35% alloys. If $x$ is the amount of 35% alloy, then it contains $0.35x$ grams of silver. The amount of 80% alloy must be $400 - x$, since the two alloys must combine to give us 400 grams. The amount of silver in this alloy is thus $0.80(400 - x)$ grams.




An expression that describes the amount of silver in each of the alloys is $0.35x + 0.80(400 - x) = 200$. This is all illustrated in the figure. We can find $x$, the amount of the 35% alloy, by solving the equation

$$0.35x + 0.8(400 - x) = 200$$
$$0.35x + 320 - 0.8x = 200$$
$$-0.45x = -120$$
$$x = \frac{-120}{-0.45} \approx 266.7 \text{ grams}$$

Thus, 266.7 grams of the 35% alloy and $400 - 266.7 = 133.3$ grams of the 80% alloy must be mixed to obtain 400 grams of a 50% silver alloy. ■

## Rate problems

Another type of problem uses rates; the rate at which a person or machine works, or at which a pipe fills a pool, or at which a computer computes. Of interest to us here are situations in which the rates can be added. This is illustrated in example 2–1 F.

■ ***Example 2–1 F***

Solve.

1. One painter can paint a certain size room in 6 hours; the painter's partner requires 10 hours to do a room of the same size. How long would it take them to paint such a room, working together?

   The first painter paints at a rate of $\frac{1}{6}$th of a room per hour, and the second at a rate of $\frac{1}{10}$th of a room per hour. We assume that working together they can paint $\frac{1}{6} + \frac{1}{10} = \frac{4}{15}$ of a room per hour. Now, if we let $t$ be the time required to paint one room, we use the idea that rate · time = one (job) or $\frac{4}{15} \cdot t = 1$.

We solve for $t$.

$$\tfrac{4}{15}t = 1$$

$$(\tfrac{15}{4})(\tfrac{4}{15})t = (\tfrac{15}{4})1 \qquad \text{Multiply both members by } \tfrac{15}{4}$$

$$t = \tfrac{15}{4} = 3\tfrac{3}{4} \text{ hours}$$

Thus, it would take the painters $3\frac{3}{4}$ hours to paint the room, working together.

**Note** The rates to complete a job add, not the times needed to complete the job. This is illustrated above.

**2.** A boat travels at 20 miles per hour (mph) in still water. It travels downstream to a certain destination and back to the starting point in 3 hours. The speed of the current is 5 mph. How far downstream is the destination?

Let $x$ represent this unknown distance downstream (and upstream). Using distance = rate × time, or $d = rt$ and solving for $t$, $t = \dfrac{d}{r}$, we summarize this information in a table.

| | $d$ miles | $r$ miles per hour | $t$ hours |
|---|---|---|---|
| Downstream | $x$ | $20 + 5 = 25$ | $\dfrac{x}{25}$ |
| Upstream | $x$ | $20 - 5 = 15$ | $\dfrac{x}{15}$ |

Since total time downstream and back upstream is 3 hours, we know that

time downstream + time upstream = 3 hours

$$\frac{x}{25} + \frac{x}{15} = 3$$

$$\frac{75}{1} \cdot \frac{x}{25} + \frac{75}{1} \cdot \frac{x}{15} = 75(3) \qquad \text{Multiply by the LCD, 75}$$

$$3x + 5x = 225$$

$$x = 28\tfrac{1}{8}$$

Since $x$ represents the distance downstream, this distance is $28\frac{1}{8}$ miles. ■

## Mastery points

**Can you**

- Solve linear equations in one variable?
- Solve literal equations in one variable?
- Solve certain word problems by setting up appropriate linear equations in one variable and solving?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## *Exercise 2–1*

Solve the following linear equations[3] by specifying the solution set.

**1.** $13x = 5 - 3x$ **2.** $-12x + 4 = 8 - x$ **3.** $9 - 2x = -5(3 + x)$

**4.** $9a - 3 = 4(3 - 2a)$ **5.** $\frac{1}{4}x + 3 = \frac{3}{8}x - 8$ **6.** $-2(5 - 3x) = x - (3 + 6x)$

**7.** $\frac{2}{3}x - 3 = 4x + 1$ **8.** $\frac{2x}{5} = 3 - x$ **9.** $-5(3x - 2) + x = 0$

**10.** $\frac{x + 3}{2} = \frac{2x}{5}$ **11.** $\frac{2 - 3x}{4} = \frac{x}{2}$ **12.** $\frac{1}{2}x = 19 - 4(5 - 2x)$

**13.** $6x - 5 = x + 5(x - 1)$ **14.** $2 - \frac{3}{4}x = 5x$ **15.** $2[(3x - 1) - (5x + 2) - 4] = 2x$

**16.** $2(x + 1) = x - (3x - 1)$ **17.** $\frac{3}{4}(4x - 3) + x = -17$ **18.** $5[3 - 2(5 - x) + 2] - x = 0$

**19.** $\frac{1 - x}{4} = \frac{1 + x}{4}$ **20.** $\frac{x + 3}{4} = \frac{x + 4}{3}$

**21.** $-4[2x - 3(3x - 2) - (5 - x)] + 6(1 - x) = 0$

**22.** $4 - 2y = 4 + 2y$ **23.** $3x - 12 = 12 - 3x$ **24.** $x + 1 = x + 2$

**25.** $5(x - 3) = -(15 - 5x)$ **26.** $\frac{1}{4}x - \frac{1}{3}x = x$

Find approximate answers to the following problems. Round the answer to four digits of accuracy.

**27.** $9.6x - 2.4(3 - 1.8x) = 10.0$ **28.** $-19.6x = 4.55(3 - 2x)$

**29.** $150x - 13.8 = 0.04(1{,}500 - 1{,}417x)$ **30.** $13.5 + 1.5x - 2.3[9 - 3.25(67 - 4.25x) - 15x] = 0$

Solve the following literal equations for the variable indicated.

**31.** $V = k + gt$; for $t$ **32.** $m = -p(\ell - x)$; for $x$ **33.** $2S = 2Vt - gt^2$; for $V$

**34.** $R = W - b(2c + b)$; for $c$ **35.** $S = \frac{n}{2}[2a + (n - 1)d]$; for $a$ **36.** $P = n(P_2 - P_1) - c$; for $P_1$

**37.** $d = d_1 + (k - 1)d_2 + (j - d_2)d_3$; for $d_2$

**38.** $\frac{x + 2y}{x - 2y} = 4$; for $x$ **39.** $\frac{x + 2y}{x - 2y} = 4$; for $y$ **40.** $\frac{x + y}{x} = y$; for $y$

**41.** $2x - y = 5x + 6y$; for $x$ **42.** $3(4x - y) = 2x + y + 6$; for $y$ **43.** $V = r^2(a - b)$; for $a$

**44.** $\frac{x + y}{3} = \frac{2x - y}{5}$; for $y$ **45.** $\frac{x + y}{3} = \frac{2x - y}{5}$; for $x$ **46.** $2S = 2Vt - gt^2$; for $g$

**47.** $V = \frac{1}{3}\pi h^2(3R - h)$; for $R$ **48.** $b(y - 4) = a(x + 3)$; for $y$ **49.** $b(y - 4) = a(x + 3)$; for $x$

**50.** $T = \frac{R - R_0}{aR_0}$; for $R$ **51.** $T = \frac{aR_0}{R - R_0}$; for $R_0$ **52.** $A = \frac{1}{2}h(b_1 + b_2)$; for $b_1$

**53.** Amdahl's law is an equation used to measure efficiency of parallel algorithms in parallel processors of computers. One form is $\frac{1}{s} = f + \frac{1 - f}{p}$. Solve this equation for $f$.

**54.** Solve Amdahl's law (see problem 53) for $s$.

Solve the following interest problems.

**55.** A total of \$15,000 is invested, part at 8% and part at 6%. The income from these investments for one year is \$1,100. How much was invested at each rate?

**56.** A total of \$28,000 is invested, part at 4% and part at 8%. The income from these investments for one year is \$2,000. How much was invested at each rate?

[3] "Someone told me that each equation I included in the book would halve the sales. I therefore resolved not to have any equations at all." Stephen Hawking, talking about his wonderful book *A Brief History of Time.*

57. \$18,000 was invested; part of the investment made a 14% gain, but the rest had a 9% loss. The net gain from the investments was \$680. How much money was invested at each rate?

58. \$25,000 was invested; part of the investment made an 8% gain, but the rest had a 9% loss. The net loss from the investments was \$890. How much was invested at each rate?

$\boxed{59.}$ Two investments were made. The larger investment was at 8%, and the smaller was at 5%. The larger investment earned \$230 more than the smaller investment, and was \$1,000 more than the smaller investment. How much money was invested at each rate?

60. Two investments were made. The larger investment was at 10%, and the smaller was at 6%. The larger investment earned \$410 more than the smaller investment, and was \$500 more than the smaller investment. How much money was invested at each rate?

61. A total of \$18,000 was invested, part at 5% and part at 9%. If the income for one year from the 9% investment was \$100 less than the income from the 5% investment, how much was invested at each rate?

$\boxed{62.}$ \$6,000 has been invested at 5% interest. How much money would have to be invested at 8% so that the interest rate on the two investments would be 6%?

Solve the following mixture problems.

63. A trucking firm has two mixtures of antifreeze; one is 35% alcohol, and the other is 65% alcohol. How much of each must be mixed to obtain 80 gallons of a 50% solution?

64. A company has 2.5 tons of material that is 30% copper. How much material that is 75% copper must be mixed with this to obtain a material that is 50% copper?

$\boxed{65.}$ A company has 3,000 gallons of a 10% pesticide solution. It also can obtain as much of a 4% pesticide solution as it needs. How much of this 4% solution should it mix with the 3,000 gallons of 10% solution so that the result is an 8% solution?

66. A drug firm has on hand 400 lb of a mixture that is 3% sodium. The firm can buy as much of a 0.8% sodium mixture as it needs. It wishes to sell a mixture that is 2% sodium. How much of the 0.8% sodium mixture must it mix with the 400 lb of on-hand material to obtain a 2% mixture?

67. A drug firm has an order for 300 liters of 40% hydrogen peroxide. It only stocks 20% and 55% solutions. How much of each should be mixed to fill the order?

Solve the following rate problems.

$\boxed{68.}$ A printing press takes 35 minutes to print 5,000 flyers; a second takes 50 minutes. (a) Running together, how long would it take to print 5,000 flyers, to the nearest second? (b) 8,000 flyers?

69. A conveyor belt takes 3 hours to move 50 tons of iron ore. A newer belt takes $2\frac{1}{4}$ hours to do the same amount of work. (a) How long (to the nearest minute) would it take both belts running together to move 50 tons of iron ore? (b) 235 tons?

70. One logging crew takes 18 hours to log 2 acres; a second crew takes 14 hours for the same job. How long would it take the crews to log 2 acres working together?

71. A book of the sixth century A.D., called the *Greek Anthology,* contains the following problem: If one pipe fills a cistern in one day, a second in two days, a third in three days, and a fourth in four days, how long will it take all four running together to fill it? Solve this problem.

72. A street sweeper takes 8 hours to sweep 20 miles of street. A new model takes 6 hours to do the same thing. (a) Working together, how long would it take both machines to sweep 20 miles of street? (b) 35 miles of street?

73. An automobile can travel 200 miles in the same time that a truck can travel 150 miles. If the automobile travels at an average rate that is 15 mph faster than the average rate of the truck, find the average rate of each.

$\boxed{74.}$ A boat moves at 16 mph in still water. If the boat travels 20 miles downstream in the same time it takes to travel 14 miles upstream, what is the speed of the current?

**75.** An airplane can cruise at 300 mph in still air. If the airplane takes the same time to fly 950 miles with the wind as it does to fly 650 miles against the wind, what is the speed of the wind?

**76.** A boat travels 40 kilometers upstream in the same time that it takes to travel 60 kilometers downstream. If the stream is flowing at 6 km per hour, what is the speed of the boat in still water?

**77.** It takes one jogger 2 minutes longer to jog a certain distance than it does another. What is this distance, if the faster jogger jogs at 7 mph and the slower at 5 mph?

**78.** An individual averages 10 miles per hour riding a bicycle to deliver papers. The same individual averages 30 mph to deliver the papers by car. If it takes one-half hour less time by car, how long is the paper route?

**79.** Prove the cross-multiplication property. It states

For any real numbers $a$, $b$, $c$, $d$, $b \neq 0$ and $d \neq 0$, if $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$.

(*Hint:* Multiply both members by the least common denominator $bd$.)

## *Skill and review*

1. Solve for $x$: $(x - 2)(x + 5) = 0$.
2. Multiply: $(2x - 3)(x + 2)$.
3. Factor: $4x^2 - 16x$.
4. Factor: $4x^2 - 1$.
5. Factor: $6x^2 - 5x - 4$.
6. Simplify: $\sqrt{-20}$.
7. Simplify: $\sqrt{8 - 4(3)(-2)}$.
8. Simplify: $\dfrac{8 - \sqrt{32}}{4}$.
9. Find the area and perimeter of a rectangle whose length is 8 inches and width is 6 inches.
10. How long will it take a vehicle that is going 45 miles per hour to travel 135 miles?

## *2–2 Quadratic equations*

One printing press takes 3 hours longer than another to print 10,000 newspapers. Running together they produce the 10,000 papers in 8 hours. Find the time required for each to do this job alone.

This section shows the mathematics necessary to deal with this problem and related types of problems.

Recall from section 1–3 that a quadratic expression is an expression of the form $ax^2 + bx + c$.

**Quadratic equation**

A quadratic equation in one variable is an equation that can be put in the form

$$ax^2 + bx + c = 0$$

$a, b, c \in R$, $a > 0$.

This form is called the **standard form** for a quadratic equation.

## Solution by factoring

If the quadratic expression in a quadratic equation in standard form can be factored, the equation can be solved using the zero product property.

> **Zero product property**
> For any algebraic expressions $A$ and $B$,
> $$AB = 0 \text{ if and only if } A = 0 \text{ or } B = 0$$
> ***Concept***
> A product can be zero if and only if one of its factors is zero.

■ ***Example 2–2 A***

Solve the quadratic equations by factoring.

1. $6p^2 = 3p$
$6p^2 - 3p = 0$
$3p(2p - 1) = 0$
$3p = 0$ or $2p - 1 = 0$
$p = 0$ or $p = \frac{1}{2}$
$\{0,\frac{1}{2}\}$

2. $x^2 + \frac{14}{3}x = \frac{5}{3}$
$x^2 + \frac{14}{3}x - \frac{5}{3} = 0$
$3x^2 + 14x - 5 = 0$
$(3x - 1)(x + 5) = 0$
$3x - 1 = 0$ or $x + 5 = 0$
$x = \frac{1}{3}$ or $x = -5$
$\{-5,\frac{1}{3}\}$ ■

## Solution by extracting the roots

When $b$ in $ax^2 + bx + c = 0$ is zero, we have a simpler equation of the form $ax^2 + c = 0$. This can be solved by the method called **extracting the roots,** which uses the fact that

$$\text{if } x^2 = k, \text{ then } x = \sqrt{k} \text{ or } x = -\sqrt{k}$$

This can be abbreviated using the symbol $\pm$, which means "plus or minus," to $x = \pm\sqrt{k}$.

■ ***Example 2–2 B***

Solve $(4x - 2)^2 = 8$ by extracting the roots.

| | |
|---|---|
| $(4x - 2)^2 = 8$ | The member with the variable is a perfect square |
| $4x - 2 = \pm\sqrt{8}$ | Extract the roots |
| $4x - 2 = \pm 2\sqrt{2}$ | $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$ |
| $4x = 2 \pm 2\sqrt{2}$ | Add 2 to both members |
| $x = \dfrac{2 \pm 2\sqrt{2}}{4}$ | Divide both members by 4 |
| $= \dfrac{2(1 \pm \sqrt{2})}{4}$ | See following note |
| $= \dfrac{1 \pm \sqrt{2}}{2}$ | Reduce by 2 |
| $\left\{\dfrac{1 \pm \sqrt{2}}{2}\right\}$ or $\left\{\dfrac{1 + \sqrt{2}}{2}, \dfrac{1 - \sqrt{2}}{2}\right\}$ | Solution set |

**Note** "$2 \pm 2\sqrt{2}$" means "$2 + 2\sqrt{2}$ or $2 - 2\sqrt{2}$." Each has a common factor 2 and can be rewritten "$2(1 + \sqrt{2})$ or $2(1 - \sqrt{2})$," which can be abbreviated as "$2(1 \pm \sqrt{2})$."

## Solution by the quadratic formula

When the methods mentioned above do not apply (or even if they do), a quadratic equation can be solved using the quadratic formula.

> **The quadratic formula**
> If $ax^2 + bx + c = 0$ and $a \neq 0$, then
> $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula, developed in the seventeenth century,[4] can be used to solve any quadratic equation. The derivation of the formula is an exercise in section 4–1 after a discussion of a procedure called "completing the square." A proof that the formula gives solutions to the quadratic equation is in the exercises of this section.

The formula is applied by determining the given values of *a*, *b*, and *c* and using substitution of value (section 1–2).

■ ***Example 2–2 C***

Solve $3 + \dfrac{6}{y^2} = \dfrac{4}{y}$ using the quadratic formula.

$3y^2 + 6 = 4y$ — Multiply each term by $y^2$

$3y^2 - 4y + 6 = 0$ — Put in standard form: $a = 3$, $b = -4$, $c = 6$

$$y = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(6)}}{2(3)}$$ Substitute into the formula

$$= \frac{4 \pm 2i\sqrt{14}}{6}$$ $\sqrt{-56} = i\sqrt{56} = i\sqrt{4 \cdot 14} = 2i\sqrt{14}$

$$= \frac{2(2 \pm i\sqrt{14})}{6}$$ Factor 2 from the numerator

$$= \frac{2 \pm i\sqrt{14}}{3}$$ Reduce

$$= \frac{2}{3} \pm \frac{\sqrt{14}}{3}i$$ Standard form for complex number

$$\left\{\frac{2}{3} \pm \frac{\sqrt{14}}{3}i\right\}$$ Solution set

[4]Quadratic equations were solved in ancient civilizations, often using geometric constructions, but negative and complex roots were rejected as not being part of the real world. Algebraic formulas for solving the quadratic equation were developed by Rafael Bombelli (ca. 1526–1573).

As can be observed in the examples above, when $b^2 - 4ac < 0$ the solutions of the quadratic equation are complex. The expression $b^2 - 4ac$ is called the **discriminant** of the quadratic expression $ax^2 + bx + c$. The solutions of the quadratic equation can be categorized as follows:

| Value of discriminant | Solutions of $ax^2 + bx + c = 0$ |
|---|---|
| $b^2 - 4ac > 0$ | Two real solutions |
| $b^2 - 4ac = 0$ | One real solution |
| $b^2 - 4ac < 0$ | Two complex solutions. |

As shown in section 2–1, a graphing calculator can be used to verify numeric solutions to equations. For example, the solutions to $2x^2 = 4x - 1$ can be verified by graphing the two equations $Y_1 = 2x^2$ and $Y_2 = 4x - 1$ (see figure 2–3). Using the "trace" feature will show that these two graphs cross at the approximate $x$-values 0.31 and 1.79. The solutions are $\frac{2-\sqrt{2}}{2} \approx 0.29$ and $\frac{2+\sqrt{2}}{2} \approx 1.71$. Using the zoom feature can produce better approximations. Note that this method cannot be used to verify complex roots.

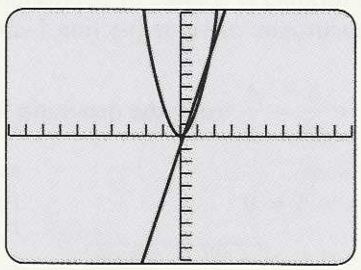


**Figure 2–3**

## General factors of a quadratic expression

In chapter 1 we reviewed factoring quadratic trinomials when the resulting factors have integer coefficients. The quadratic formula also allows us to factor *any* quadratic expression by the following theorem.

> **Factors of a quadratic expression**
>
> Given $ax^2 + bx + c$, $a \neq 0$, then
>
> $$ax^2 + bx + c = a\left(x - \frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(x - \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$

■ ***Example 2–2 D***

Factor the quadratic expression $3y^2 - 4y + 6$.

From example 2–2 C the zeros are $\frac{2}{3} \pm \frac{\sqrt{14}}{3}i$, so $3y^2 - 4y + 6$

$$= 3\left[y - \left(\frac{2}{3} + \frac{\sqrt{14}}{3}i\right)\right]\left[y - \left(\frac{2}{3} - \frac{\sqrt{14}}{3}i\right)\right].$$

## Some geometry

A **right triangle** is a triangle with one right (90°) angle (see figure 2–4). The side opposite that angle is called the **hypotenuse** and is always the longest side of the triangle. If $c$ represents the length of the hypotenuse, and $a$ and $b$ represent the lengths of the remaining two sides, then the following relation holds: $a^2 + b^2 = c^2$. This is the **Pythagorean theorem.**[5]

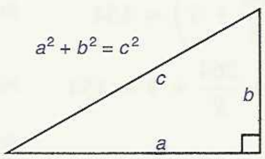



**Figure 2–4**

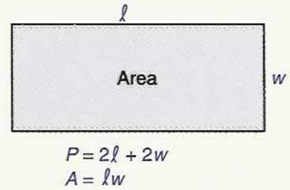
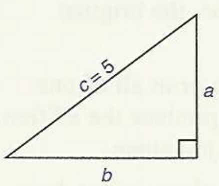



**Figure 2–5**

A **rectangle** is a four-sided figure with four right angles (see figure 2–5). The lengths of its sides are called the length and width. The **perimeter** is the distance around the figure; the **area** of the figure describes the size of its surface. If $\ell$ means length, $w$ means width, $P$ means perimeter, and $A$ means area then the following formulas hold for rectangles: $P = 2\ell + 2w$ and $A = \ell w$.

### ■ *Example 2–2 E*




Solve the following problems.

**1.** In the right triangle in the figure the length $c$ of the hypotenuse is 5 and length $b$ is 1 unit larger than length $a$. Find $a$ and $b$.

In this triangle we are told that side $b$ can be described as $a + 1$. We begin with the Pythagorean theorem, which we know to be true for any right triangle.

| | |
|---|---|
| $a^2 + b^2 = c^2$ | Pythagorean theorem |
| $a^2 + (a + 1)^2 = 5^2$ | $b = a + 1$ |
| $a^2 + (a^2 + 2a + 1) = 25$ | Perform indicated operations |
| $2a^2 + 2a - 24 = 0$ | Put in standard form |
| $a^2 + a - 12 = 0$ | Divide each member by 2 |
| $(a + 4)(a - 3) = 0$ | Factor |
| $a + 4 = 0$ or $a - 3 = 0$ | Zero product property |
| $a = -4$ or $a = 3$ | Solve each linear equation |

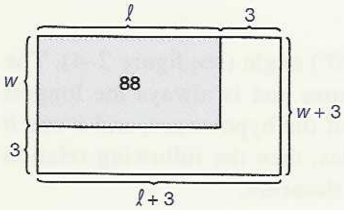
The value of $a$ cannot be $-4$ since it represents the length of the side of a triangle. Thus, $a = 3$.

We now find $b$.

$$\begin{aligned} b &= a + 1 \\ &= 3 + 1 = 4 \end{aligned}$$

Thus $a = 3$ and $b = 4$.

[5]Named for the Greek mathematician Pythagoras of Samos (ca. 580–500 B.C.), but known to the Mesopotamians 4,000 years ago.




**2.** The area of a rectangle is 88 in.$^2$. If the length and width are each increased by 3 inches the new area is 154 in.$^2$ (see the figure). Find the original length and width, $\ell$ and $w$.

| | |
|---|---|
| $\ell w = 88$ | Area of original figure |
| $w = \dfrac{88}{\ell}$ | Solve for $w$ |
| $(\ell + 3)(w + 3) = 154$ | Area of new, larger figure |
| $(\ell + 3)\left(\dfrac{88}{\ell} + 3\right) = 154$ | Replace $w$ by $\dfrac{88}{\ell}$ |
| $88 + 3\ell + \dfrac{264}{\ell} + 9 = 154$ | Perform the multiplication |
| $3\ell + \dfrac{264}{\ell} - 57 = 0$ | Add $-154$ to both members; combine like terms |
| $3\ell^2 - 57\ell + 264 = 0$ | Multiply both members by $\ell$ |
| $\ell^2 - 19\ell + 88 = 0$ | Divide both members by 3 |
| $(\ell - 8)(\ell - 11) = 0$ | Factor the quadratic trinomial |
| $\ell - 8 = 0$ or $\ell - 11 = 0$ | Use the zero product property |
| $\ell = 8$ or $\ell = 11$ | Solve each linear equation |

Since $w = \dfrac{88}{\ell}$, we obtain the solutions $w = 11$ when $\ell = 8$, or $w = 8$ when $\ell = 11$. We select the second solution, since we usually think of length as greater than width. (Either solution is valid; they are just different ways to label the same geometric figure.) Thus, the original length is 11 inches and the original width is 8 inches.

**3.** One printing press takes 2 hours longer than another to print all of one edition of a certain newspaper. Running together they produce the edition in $1\frac{1}{3}$ hours. Find the time required for each to do this job alone.

This type of problem requires the principle that rate × time = part done. In this problem the part done is one edition of the newspaper, represented by 1. Thus we use rate × time = 1. Observe that this means that rate $= \dfrac{1}{\text{time}}$, which we use below.

If it takes $x$ hours for the faster machine to print the edition, then its rate is $\dfrac{1}{x}$ papers per hour $\left(\text{rate} = \dfrac{\text{one job}}{\text{time}}\right)$. The slower machine takes $x + 2$ hours, so its rate is $\dfrac{1}{x + 2}$ papers per hour. Their combined rate is $\dfrac{1}{x} + \dfrac{1}{x + 2}$.

It takes $1\frac{1}{3}$ hours to print the edition when the presses are running together, so the combined rate is $\frac{1}{\text{time}} = \frac{1}{1\frac{1}{3}} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$. Thus, we know that $\frac{1}{x} + \frac{1}{x+2} = \frac{3}{4}$, which we solve for $x$.

$$\frac{1}{x} + \frac{1}{x+2} = \frac{3}{4}$$

$$\frac{4x(x+2)}{1} \cdot \frac{1}{x} + \frac{4x(x+2)}{1} \cdot \frac{1}{x+2} = \frac{4x(x+2)}{1} \cdot \frac{3}{4}$$

Multiply by the LCD, $4x(x + 2)$

$$4(x+2) + 4x = 3x(x+2)$$

$$8x + 8 = 3x^2 + 6x$$

$$3x^2 - 2x - 8 = 0$$

Perform indicated operations, combine like terms, and put in standard form

$$(3x+4)(x-2) = 0$$

Factor

$$3x + 4 = 0 \text{ or } x - 2 = 0$$

Zero product property

$$x = -\frac{4}{3} \text{ or } x = 2$$

$$x = 2$$

We reject the negative value; it could not describe the running time for the presses

Thus the times for the presses running alone are $x = 2$ and $x + 2 = 4$, or 2 hours and 4 hours.

**4.** When flying directly into a 20 mph wind it takes an aircraft $1\frac{1}{2}$ hours longer to travel 400 miles than when there is no wind. Find the time required for the aircraft to fly this distance in no wind.

We use the relation rate × time = distance, or $rt = d$. Let $x$ be the desired value—the time for the aircraft to fly 400 miles in no wind. Then $x + 1\frac{1}{2}$ is the time when there is a wind. We can show this information graphically with a table. Since $r = \frac{d}{t}$, the rate for a trip under no wind and wind conditions is $\frac{400}{x}$ and $\frac{400}{x + 1\frac{1}{2}}$.

| | $r = \frac{d}{t}$ | $t$ | $d$ |
|---|---|---|---|
| Into wind | $\frac{400}{x+1\frac{1}{2}}$ | $x + 1\frac{1}{2}$ | 400 |
| No wind | $\frac{400}{x}$ | $x$ | 400 |

The wind slows down the aircraft by the speed of the wind, so we know that $\frac{400}{x} - \frac{400}{x + 1\frac{1}{2}} = 20$. We can now solve for $x$.

$$\frac{20}{x} - \frac{20}{x + \frac{3}{2}} = 1$$ Divide each term by 20

$$\frac{20}{x} - \frac{2}{2} \cdot \frac{20}{x + \frac{3}{2}} = 1$$ Multiply the second term by $\frac{2}{2}$

$$\frac{20}{x} - \frac{40}{2x + 3} = 1$$

$$\frac{x(2x + 3)}{1} \cdot \frac{20}{x} - \frac{x(2x + 3)}{1} \cdot \frac{40}{2x + 3} = x(2x + 3)(1)$$ Multiply by the LCD, $x(2x + 3)$

$$20(2x + 3) - 40x = x(2x + 3)$$

$$2x^2 + 3x - 60 = 0$$ Perform operations and put in standard form

$$x = \frac{-3 \pm \sqrt{9 - 4(2)(-60)}}{2(2)} = \frac{-3 \pm \sqrt{489}}{4}$$

We choose the positive value for $x$, obtaining $\frac{-3 + \sqrt{489}}{4} \approx 4.8$ hours for the aircraft to fly 400 miles in no wind conditions. ■

## The domain of a rational expression

One application of solving linear and quadratic equations is determining when a rational expression is not defined. Recall that section 1–4 introduced rational expressions. All values for which an expression is defined are called its domain.

> **Domain of an expression**
> The domain of an expression is the set of all replacement values of the variable for which the expression is defined.

In the case of rational expressions the denominator presents a special problem. Division by zero is not defined; thus, the domain of a rational expression must exclude all values that cause the denominator to have the value 0. For example, in the expression $\frac{2x - 1}{x - 3}$, we cannot permit $x$ to take on the value 3, since this would evaluate to $\frac{2(3) - 1}{3 - 3} = \frac{5}{0}$, and this expression is not defined. We would express the domain of this expression as $\{x \mid x \neq 3\}$. *To determine which values to exclude we write an equation in which one side is the denominator and the other is zero and solve.* The solutions must be excluded from the domain.

■ *Example 2–2 F*

Determine the domain of the following rational expressions.

**1.** $\dfrac{x-3}{2x+5}$

$2x + 5 = 0$ — Set the denominator equal to zero and solve

$2x = -5$ — Add −5 to both members (sides) of the equation

$x = -\frac{5}{2}$ — Divide both members by 2

Thus the domain is $\{x \mid x \neq -\frac{5}{2}\}$. — All real numbers except $-\frac{5}{2}$

**Note** We do not concern ourselves with zeros of the numerator. This is because division *into* zero is defined.

**2.** $\dfrac{x-3}{x^2 - 3x - 10}$

$x^2 - 3x - 10 = 0$ — Set the denominator equal to 0; the result is a quadratic equation

$(x - 5)(x + 2) = 0$ — Factor the quadratic expression

$x - 5 = 0$ or $x + 2 = 0$ — Set each factor to 0

$x = 5$ or $x = -2$

Thus the domain is $\{x \mid x \neq 5, x \neq -2\}$. — All real numbers except −2 and 5

**3.** $\dfrac{3z}{z^2 + 4}$

We know that $z^2 \geq 0$, so that $z^2 + 4$ cannot ever be zero (it cannot be less than 4). Thus, $z$ can take on any value, and the domain is $R$ (the set of real numbers). ■

## Solution using substitution

The expressions in many equations are quadratic, not in a simple variable such as $x$, but in some more complicated variable expression, such as $x^2$, $x^{-1}$, $\sqrt{x}$, or $(x + 1)^2$. For example, the following expressions are quadratic in the variable expression shown.

| Expression | Variable expression | Quadratic form of the equation |
|---|---|---|
| $3x^2 - 2x - 5$ | $x$ | $3x^2 - 2x - 5$ |
| $3x^4 - 2x^2 - 5$ | $x^2$ | $3(x^2)^2 - 2(x^2) - 5$ |
| $3x^{-2} - 2x^{-1} - 5$ | $x^{-1}$ | $3(x^{-1})^2 - 2(x^{-1}) - 5$ |
| $3x - 2\sqrt{x} - 5$ | $\sqrt{x}$ | $3(\sqrt{x})^2 - 2(\sqrt{x}) - 5$ |
| $3(x + 1)^2 - 2(x + 1) - 5$ | $(x + 1)$ | $3(x + 1)^2 - 2(x + 1) - 5$ |

The technique of substitution for expression (section 1–3) can help solve equations that are quadratic in more complicated expressions like those above.

**Solving quadratic equations using substitution for expression**

1. Determine the variable expression; this is usually the variable factor with the exponent with the smallest absolute value.
2. Let $u$ represent the variable expression. Calculate $u^2$.
3. Substitute $u$ and $u^2$ as appropriate.
4. Solve the quadratic equation for $u$.
5. Substitute the variable expression for $u$ in the solutions.
6. Solve these equations for the variable.

## ■ *Example 2–2 G*

Solve each equation.

**1.** $3x^{-2} - 2x^{-1} - 5 = 0$ — Step 1: The variable factor with exponent of smallest absolute value is $x^{-1}$

Step 2: Let $u = x^{-1}$; then $u^2 = x^{-2}$

$3u^2 - 2u - 5 = 0$ — Step 3: Replace $u$ by $x^{-1}$, $x^{-2}$ by $u^2$

$u = \dfrac{5}{3}$ or $u = -1$ — Step 4: Solve the quadratic equation for $u$

$x^{-1} = \dfrac{5}{3}$ or $x^{-1} = -1$ — Step 5: Replace $u$ by $x^{-1}$

$\dfrac{1}{x} = \dfrac{5}{3}$ or $\dfrac{1}{x} = -1$ — Step 6: Solve; recall that $x^{-1} = \dfrac{1}{x}$

$x = \frac{3}{5}$ or $x = -1$

$\{-1, \frac{3}{5}\}$ — Solution set

**2.** $3x - 2\sqrt{x} - 5 = 0$ — Let $u = \sqrt{x}$; then $u^2 = x$

$3u^2 - 2u - 5 = 0$ — Replace $\sqrt{x}$ by $u$

$u = \frac{5}{3}$ or $u = -1$ — Solve the quadratic equation for $u$

$\sqrt{x} = \frac{5}{3}$ or $\sqrt{x} = -1$ — Replace $u$ by $\sqrt{x}$

$x = \frac{25}{9}$ — Square both members to obtain $x$

**$\sqrt{x} = -1$ does not yield a solution. The square root of any number, real or complex, cannot be a negative real number.**

$\{\frac{25}{9}\}$ — Solution set ■

### Mastery points

**Can you**

- Solve certain quadratic equations by factoring?
- Solve equations of the form $ax^2 + c = 0$ by extracting the roots?
- Solve any quadratic equation by using the quadratic formula?
- Solve certain word problems that involve quadratic equations?
- Recognize and solve equations that are of quadratic type using substitution of expression?
- Determine the domain of a rational expression?

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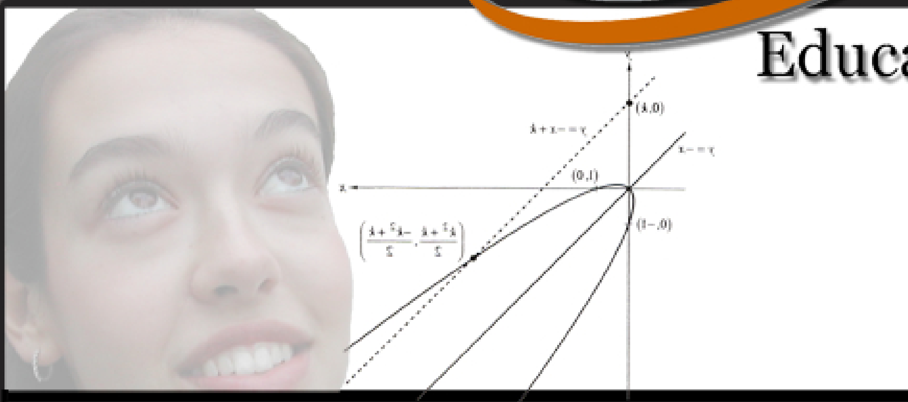

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## Exercise 2–2

Solve the following quadratic equations for $x$ by factoring.

**1.** $x^2 = 7x + 8$
**2.** $y^2 - 18y + 81 = 0$
**3.** $q^2 = 5q$
**4.** $2x^2 - 3x - 2 = 0$
**5.** $\frac{x^2}{6} = \frac{x}{3} + \frac{1}{2}$
**6.** $x - 1 = \frac{x^2}{4}$
**7.** $3x^2 + 5x - 2 = 0$
**8.** $6x^2 = 20x$
**9.** $\frac{x}{2} + \frac{7}{2} = \frac{4}{x}$
**10.** $(p + 4)(p - 6) = -16$
**11.** $(3x + 2)(x - 1) = 7 - 7x$
**12.** $x^2 - 4ax + 3a^2 = 0$
**13.** $5x^2 - 6y^2 = 7xy$
**14.** $3x^2 - 13xy + 4y^2 = 0$
**15.** $12x^2 = 8ax + 15a^2$
**16.** $12xy - 6y^2 = 6x^2$

Solve the following equations by extracting the roots.

**17.** $3y^2 = 27$
**18.** $3y^2 - 72 = 0$
**19.** $m^2 - 40 = 0$
**20.** $x^2 = 10$
**21.** $9x^2 - 40 = 0$
**22.** $4x^2 = 25$
**23.** $5x^2 = 32$
**24.** $7x^2 - 20 = 10$
**25.** $(x - 3)^2 = 10$
**26.** $(2x - 1)^2 + 6 = 0$
**27.** $3(x + 1)^2 = 8$
**28.** $5(2x + 5)^2 + 8 = 0$
**29.** $a(bx + c)^2 = d$, assuming $a, d > 0$
**30.** $(y - a)^2 - b^2 = 0, b > 0$

Solve the following quadratic equations using the quadratic formula.

**31.** $3y^2 - 5y = 6$
**32.** $3x - 7 = 5x^2$
**33.** $2y^2 + \frac{4y}{3} = 5$
**34.** $3x - \frac{3}{x} = 8$
**35.** $(z + 1)(z - 1) = 6z + 4$
**36.** $\frac{1}{x + 2} - x = 5$

Factor the following quadratic expressions.

**37.** $5x^2 - 8x - 12$
**38.** $3x^2 + 2x - 9$
**39.** $2x^2 + 6x - 4$
**40.** $x^2 - x - 6$

Solve the following problems.

**41.** In a right triangle the length of the hypotenuse is 20, and one side is four units longer than the other. Find the length of the longer side.

**42.** In a right triangle one of the sides is five units longer than the other. The length of the hypotenuse is 50. Find the length of the shorter side.

**43.** A rectangular playground has a length 5 feet less than three times the width, and the distance from one corner to the opposite corner is 100 feet. Find the dimensions of the playground.

**44.** A rectangle that is 6 centimeters long and 3 centimeters wide has its dimensions increased by an equal amount. The area of this new rectangle is three times that of the old rectangle. What are the dimensions of the new rectangle?

**45.** The area of a rectangular floor is 1,196 square meters. The width is 3 meters less than half the length. Find the dimensions of the floor.

**46.** A cuneiform tablet from Mesopotamia, thousands of years old, asks for the solution to the system $xy = 7\frac{1}{2}$, $x + y = 6\frac{1}{2}$. By the first equation $y = \frac{15}{2x}$, so the second equation is $x + \frac{15}{2x} = 6\frac{1}{2}$. Solve this new equation for $x$, then find the value of $y$.

**47.** The demand equation for a certain commodity is given by $D = \frac{2{,}000}{p}$, where $D$ is the demand for the commodity at price $p$ dollars per unit. The supply equation for the commodity is $S = 300p - 400$, where $S$ is the quantity of the commodity that the supplier is willing to supply at $p$ dollars per unit. Find the equilibrium price (where supply equals demand).

**48.** A manufacturer finds that the total cost $C$ for a solar energy device is expressed by $C = 50x^2 - 24{,}000$, and the total revenue $R$ at a price of \$200 per unit to be $R = 200x$, where $x$ is the number of units sold. What is the break even point (where total cost = total revenue)?

**49.** One printing press takes 3 hours longer than another to print 10,000 newspapers. Running together they produce the 10,000 papers in 8 hours. Find the time required for each to do this job alone.

**50.** Two pipes can be used to fill a swimming pool. When both pipes run together it takes 18 hours to fill the pool. When they run separately it takes the smaller pipe 2 hours longer to fill the pool than the larger pipe. Find the time required for the larger pipe to fill the pool alone.

**51.** When flying directly into a 15 mph wind it takes an aircraft 30 minutes longer to travel 600 miles than when there is no wind. Find the time required for the aircraft to fly this distance in no wind.

**52.** A certain river boat makes 75-mile trips up and down a river. The river flows at 4 mph. It takes the boat 7 hours longer to make the trip upstream than downstream. Find the time it would take the boat to make the trip in still water, to the nearest 0.1 hours.

Determine the domain of the following rational expressions. State the domain in set-builder notation.

**53.** $\dfrac{x-2}{2x-3}$ **54.** $\dfrac{x+1}{x}$ **55.** $\dfrac{5z}{2z+4}$ **56.** $\dfrac{2x+3}{x^2-3x}$

**57.** $\dfrac{3m-1}{3m^2-6m}$ **58.** $\dfrac{4x+1}{x^2+8x+16}$ **59.** $\dfrac{2-3x}{x^2-4x-21}$ **60.** $\dfrac{2x+9}{x^2-9}$

**61.** $\dfrac{x^2}{x^2+4}$ **62.** $\dfrac{4x^2-2x+1}{x^2+1}$

Solve the following quadratic equations using substitution.

**63.** $x^4+3x^2-28=0$ **64.** $x^6+3x^3-28=0$ **65.** $(x-3)^2-4(x-3)-9=0$

**66.** $(m+3)^2+3(m+3)=20$ **67.** $4t-3\sqrt{t}=8$ **68.** $3x-\sqrt{x}=2$

**69.** $x^{2/3}-10x^{1/3}=-9$ **70.** $3y^{1/3}+5y^{1/6}-8=0$ **71.** $4y^{-4}+4=17y^{-2}$ **72.** $x^{-4}=5x^{-2}-4$

**73.** Show by direct substitution of each value for $x$ that both $\dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $\dfrac{-b-\sqrt{b^2-4ac}}{2a}$ are solutions to the equation $ax^2+bx+c=0$ if $a \neq 0$.

**74.** Show by multiplication that, if $a \neq 0$, then

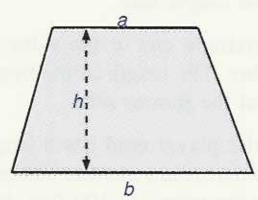
$$a\left(x-\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(x-\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$
$$= ax^2+bx+c$$

**75.** Find a complex number $a+bi$ so that $(a+bi)^2 = -5-12i$.

**76.** Find a complex number $a+bi$ so that $(a+bi)^2 = 7-24i$.

**77.** Find expressions for $a$ and $b$ (in terms of $c$ and $d$) so that, for any complex number $c+di$, $(a+bi)^2 = c+di$. Make sure that the values for $a$ and $b$ are real, and not themselves complex.

**78.** The *Sulvasutras* (800 B.C.) is an ancient Hindu manual on geometry. In it appears the problem of the Great Altar (the Maha-Vedi).[6] The altar is trapezoidal in shape with the dimensions $a = 24$, $b = 30$, and $h = 36$. The area of a trapezoid is $A = h\left(\dfrac{a+b}{2}\right)$, so in this case the area was $36\left(\dfrac{24+30}{2}\right) = 972$ square units.




The area of the altar was to be increased by $m$ square units to $972+m$. The dimensions are to be increased proportionately by $p$ units, to $24p$, $30p$, and $36p$.

**a.** Show that the new area will be $972p^2$ units.

**b.** Use the fact that $972p^2 = 972+m$ to find the new dimensions of an altar that is 100 square units larger than the original (i.e., $m$ is 100).

[6]From "Hindu Romance with Quadratic Equations" by Dr. Gurcharan Singh Bhalla, *The Amatyc Review*, Fall, 1987.

### *Skill and review*

1. Compute: $(3\sqrt{2x})^2$.
2. Compute: $(3 + \sqrt{2x})^2$.
3. Solve: $\sqrt{x} = 4$.
4. Solve: $\sqrt[3]{x} = 4$.
5. $\sqrt{\frac{2}{3} + 2} =$ **a.** $\frac{2}{3}$ **b.** $\frac{2\sqrt{2}}{3}$ **c.** $\frac{2\sqrt{3}}{3}$ **d.** $\frac{2\sqrt{6}}{3}$

## *2–3 Equations involving radicals*

At an altitude of $h$ feet above level ground the distance $d$ in miles that a person can see an object is given by $d = \sqrt{\dfrac{3h}{2}}$. How many feet up must a person be to see an object that is 8 miles away?

This section discusses how to deal with the type of equation in this problem.

Equations with at least one term containing a radical expression involving a variable are solved using the following principle.

**Property of $n$th power**

If $n \in N$ and $P$ and $Q$ are algebraic expressions, then all of the solutions of the equation $P = Q$ are also solutions of $P^n = Q^n$.

This says that we can solve equations by raising both members to the same positive integer power $n$. It also implies that *we may get solutions that are not solutions to the original equation.* In particular, this can happen when $n$ is even, but not odd. For example,

| | | |
|---|---|---|
| $x = -2$ | $x = -2$ | Original equation |
| $x^2 = (-2)^2$ | $x^3 = (-2)^3$ | Raise both members to a power |
| $x^2 = 4$ | $x^3 = -8$ | New equation |

$x^2 = 4$ has two solutions, $\pm 2$, whereas the solution to the original equation is only $-2$. $x^3 = -8$ has only the single real solution $-2$.

These extra solutions (such as 2 in $x^2 = 4$, above) are called **extraneous roots.** The best way to detect this situation is to *check all solutions in the original equation when we raised both members of an equation to an even power.* Of course it is always good practice to check any solution, and we shall do so in the following examples.

## Raising radical expressions to a power

We will use the fact that $(\sqrt{x})^2 = x$, $(\sqrt[3]{x})^3 = x$, $(\sqrt[4]{x})^4 = x$, and so on quite a bit. If a member of an equation has a factor that is a radical, we will raise both members of the equation to the power that corresponds to the index of the radical. This eliminates the radical.

Example 2–3 A illustrates some of the algebra we will encounter.

### ■ Example 2–3 A

Perform the indicated operations.

1. $(\sqrt{x-2})^2$ — One term being squared
$= x - 2$

2. $(\sqrt{x} - 2)^2$ — Square a two-termed expression
$= (\sqrt{x} - 2)(\sqrt{x} - 2)$ — $(a-b)^2 = (a-b)(a-b)$
$= \sqrt{x}\sqrt{x} - 2\sqrt{x} - 2\sqrt{x} + 4$ — $(a-b)(a-b) = a^2 - ab - ab + b^2$
$= x - 4\sqrt{x} + 4$ — Combine like terms

3. $(2\sqrt{3x}\sqrt{x-2})^2$ — Square a one-termed expression
$= 2^2(\sqrt{3x})^2(\sqrt{x-2})^2$ — $(ab)^n = a^nb^n$
$= 4(3x)(x-2)$
$= 12x^2 - 24x$

## Solutions of equations by raising each member to a power

Example 2–3 B illustrates how to solve equations involving radicals by raising both members to a power.

### ■ Example 2–3 B

Solve the equations.

1. $x - \sqrt{x+12} = 0$
$x = \sqrt{x+12}$ — Put the radical alone in one member of the equation before proceeding
$x^2 = (\sqrt{x+12})^2$ — Square both members
$x^2 = x + 12$
$x^2 - x - 12 = 0$ — Standard form for a quadratic equation
$(x-4)(x+3) = 0$ — Factor the quadratic trinomial
$x = 4$ or $-3$

Check the solutions: $4 = \sqrt{4+12}$
$4 = \sqrt{16}$ — True; the value 4 checks in the original equation
$-3 = \sqrt{-3+12}$
$-3 = \sqrt{9}$ — False; the value $-3$ does not check in the original equation, so it is not a solution

$\{4\}$ — Solution set

2. $\sqrt[3]{5x-2} + 3 = 0$
$\sqrt[3]{5x-2} = -3$ — Put the radical alone in one member by adding $-3$ to both members
$(\sqrt[3]{5x-2})^3 = (-3)^3$ — Raise both members to the value of the index, 3
$5x - 2 = -27$
$x = -5$

Check: $\sqrt[3]{5(-5)-2} + 3 = 0$
$\sqrt[3]{-27} + 3 = 0$
$-3 + 3 = 0$ — True, so $-5$ is a solution to the original equation

$\{-5\}$ — Solution set

**3.** $\sqrt{3x+1} + \sqrt{x-1} = 6$

It is a good idea to not square with two radicals on one side of an equation. *Rewrite the equation so that one radical is the only term on one side.*

| | |
|---|---|
| $\sqrt{3x+1} = 6 - \sqrt{x-1}$ | A radical is the only term in the left member of the equation |
| $(\sqrt{3x+1})^2 = (6 - \sqrt{x-1})^2$ | Square both members |
| $3x + 1 = (6 - \sqrt{x-1})(6 - \sqrt{x-1})$ | Rewrite the right member as a product, for convenience |
| $3x + 1 = 36 - 12\sqrt{x-1} + (x - 1)$ | Multiply |
| $2x - 34 = -12\sqrt{x-1}$ | Make the radical the only term in one member before squaring again |
| $x - 17 = -6\sqrt{x-1}$ | Divide each term by 2 |
| $(x - 17)^2 = (-6\sqrt{x-1})^2$ | Square both members again |
| $(x - 17)(x - 17) = (-6)^2(\sqrt{x-1})^2$ | |
| $x^2 - 34x + 289 = 36(x - 1)$ | |
| $x^2 - 70x + 325 = 0$ | Standard form |
| $(x - 5)(x - 65) = 0$ | Factor |
| $x = 5$ or $65$ | Possible solutions |
| $\{5\}$ | Solution set; 65 does not check |

### Mastery points

**Can you**

- Compute powers of certain expressions?
- Solve certain equations involving radicals by raising both members to powers as necessary?

## *Exercise 2–3*

Perform the indicated operations.

**1.** $(\sqrt{2x+3})^2$ **2.** $(\sqrt{5-x})^2$ **3.** $(\sqrt[3]{4x+3})^3$ **4.** $(\sqrt[3]{x+11})^3$

**5.** $(\sqrt{x}-5)^2$ **6.** $(\sqrt{3x}-1)^2$ **7.** $(3\sqrt{x}+2)^2$ **8.** $(2\sqrt[3]{3-5x})^3$

**9.** $(\sqrt{2x}-2)^2$ **10.** $(1-\sqrt{x})^2$ **11.** $(\sqrt{x-1}-2)^2$ $\boxed{\textbf{12.}}$ $(x+\sqrt{x+1})^2$

**13.** $(2\sqrt{x-1})^2$ **14.** $(\sqrt{2x-1}-1)^2$ **15.** $(\sqrt{x+3}+3)^2$ **16.** $(3-\sqrt{x-1})^2$

**17.** $(1-\sqrt{1-x})^2$ **18.** $(\sqrt{3x-2}-2)^2$

Find the solution set for each equation.

**19.** $\sqrt{2x} = 8$ **20.** $\sqrt[3]{4x} = -2$ **21.** $\sqrt[3]{\dfrac{x}{2}} = 3$ **22.** $\sqrt{\dfrac{2x}{3}} = 5$

**23.** $\sqrt{3-x} = -2$ **24.** $\sqrt[3]{3-x} = -2$ **25.** $\sqrt[5]{x+3} = -1$ **26.** $\sqrt[4]{x+3} = -1$

27. $\sqrt{2 - 3x} - 5 = 0$
28. $\sqrt[3]{x + 2} + 2 = 0$
29. $\sqrt[3]{2x + 3} + 5 = 0$
30. $\sqrt[4]{2y - 3} = 2$
31. $\sqrt[3]{2 - 3x} = -4$
32. $\sqrt[5]{2 - 6x} = -2$
33. $\sqrt[4]{x^2 - 24x} = 3$
34. $\sqrt{\dfrac{2x + 1}{2}} - 4 = 0$
35. $\sqrt{w^2 - 6w} = 4$
36. $\sqrt{x^2 + 6x + 9} - 6 = 0$
37. $\sqrt{x^2 - 5x + 2} - 4 = 0$
38. $\sqrt{x^2 - x + 6} + 6 = 0$
39. $\sqrt{5x + 1} - \sqrt{11} = 0$
40. $\sqrt{9a + 5} = \sqrt{3a - 1}$
41. $\sqrt{2p + 5} - \sqrt{3p + 4} = 0$
42. $2\sqrt{2z - 1} - \sqrt{3z} = 0$
43. $\sqrt{m}\sqrt{m - 8} = 3$
44. $\sqrt{x}\sqrt{x - 2} = 2\sqrt{6}$
45. $\sqrt{y}\sqrt{y - 5} - 6 = 0$
46. $\sqrt{2m}\sqrt{m + 2} - 4 = 0$
47. $\sqrt{u - 1} = u - 3$
48. $\sqrt{3x + 10} - 3x = 4$
49. $\sqrt{x - 2} = x - 2$
50. $\sqrt{2a} = 4 - a$
51. $\sqrt{2n + 3} - \sqrt{n - 2} = 2$
52. $3 - \sqrt{y + 4} = \sqrt{y + 7}$
53. $\sqrt{p + 1} = \sqrt{2p + 9} - 2$
54. $\sqrt{y + 4} + \sqrt{y + 7} = 3$
55. $(2y + 3)^{1/2} - (4y - 1)^{1/2} = 0$
56. $(1 - 2y)^{1/2} + (y + 5)^{1/2} = 4$
57. $(4x + 2)^{1/2} - (2x)^{1/2} = 0$
58. $(x - 2)^{1/2} = (5x + 1)^{1/2} - 3$

Solve the following formulas for the indicated variable.

59. $\sqrt{\dfrac{y^2 - x^2}{3}} = y - 3$; for $x$
60. $r = \sqrt{\dfrac{A}{\pi} - R^2}$; for $A$
61. $D = \sqrt[3]{\dfrac{6A}{\pi}}$; for $A$
62. $v = \sqrt{\dfrac{2gKE}{W}}$; for $W$
63. $\sqrt{s - t} = t + 3$; for $s$
64. $4x\sqrt{xy} = 3$; for $y$

65. On wet pavement the velocity $V$ of a car, in miles per hour, which skids to a stop is approximated by $V = 2\sqrt{3S}$, where $S$ is the length of the skid marks in feet. How long would the skid marks be if a car was traveling at 60 mph when the brakes were applied?

66. At an altitude of $h$ feet above level ground the distance $d$ in miles that a person can see an object is given by $d = \sqrt{\dfrac{3h}{2}}$. How many feet up must a person be to see an object that is 8 miles away?

67. The volume of a sphere is related to its radius by $r = \sqrt[3]{\dfrac{3V}{4\pi}}$. Solve this for $V$.

68. If a falling object falls a distance $s$ in $t$ seconds then $t = \sqrt{\dfrac{2s}{g}}$. Solve this for $s$.

69. Assume a bank account pays a simple yearly interest rate $i$, compounded every six months. If $A$ is the amount in the account after one year, on a deposit $P$, then $i = 2\left(\sqrt{\dfrac{A}{P}} - 1\right)$ is true. Show that $A = P\left(1 + \dfrac{i}{2}\right)^2$.

70. Use the formula of problem 69 to find the simple yearly interest rate $i$ if a deposit earns \$324 on a deposit of \$4,500 after one year. (This means that $A$ is \$4,500 + \$324 = \$4,824.)

## Skill and review

1. Is the statement $3(2x + 1) > x + 6$ true for $x = \frac{1}{2}$?
2. Graph the interval $-2 < x < 3$.
3. Is the statement $\frac{1}{2} < \frac{1}{3}$ true?
4. Is the statement $-\frac{1}{2} < -\frac{1}{3}$ true?
5. Is the statement $\dfrac{x}{x - 3} > x$ true for $x = -2$?
6. Which of the following are true statements?
   **a.** $5 > 2$
   **b.** $5 + 3 > 2 + 3$
   **c.** $5 - 3 > 2 - 3$
   **d.** $3(5) > 3(2)$
   **e.** $\dfrac{5}{3} > \dfrac{2}{3}$
   **f.** $(-1)(5) > (-1)(2)$

## 2–4 *Inequalities in one variable*

> The zoning bylaws of Carlisle, Massachusetts, require, among other things, that building lots have a ratio of area $A$ to perimeter $P$ conforming to the relation $\frac{16A}{P^2} > 0.4$.

As in this example, an inequality is a statement that two expressions are related by order; that is, that one is greater or less than another. For example, $5x > 10$ is an inequality that states that $5x$ is greater than 10, or equivalently, $10 < 5x$ (10 is less than $5x$). Recall from section 1–1 that a simple linear inequality can be graphed as an interval.

Any statement involving the symbols $<$, $>$, $\leq$, and $\geq$ is an inequality. Some are linear, and some are nonlinear. We will examine **linear** and **nonlinear inequalities.** Nonlinear inequalities involve expressions in which some of the variables have exponents other than one or are in denominators. Examples of each are

| **Linear** | | **Nonlinear** | |
|---|---|---|---|
| $5x < 4$ | | $5x^2 < 4$ | Variable with exponent |
| $\frac{x}{3} \geq 9$ | Number in denominator | $\frac{3}{x} \geq 9$ | Variable in denominator |
| $2x - 3 > 4x + 9$ | | $3x^2 - 2x > 9$ | |
| $x - 4 > 0$ | | $(x - 4)(x + 3) > 0$ | Variable with exponent, if we multiply |

## Linear inequalities

Algebraic methods for solving linear inequalities in one variable are much the same as those for linear equations. They use the following properties.

> **Addition property of inequality**
> For any algebraic expressions $A$, $B$, and $C$,
> if $A < B$ then $A + C < B + C$
>
> **Multiplication property of inequality**
> For any algebraic expressions $A$, $B$, and $C$,
> 1. if $C > 0$ and $A < B$ then $AC < BC$
> 2. if $C < 0$ and $A < B$ then $AC > BC$.

Thus we solve linear inequalities exactly the same way as linear equations with one exception: *if we multiply (or divide) both members of an inequality by a negative value we must reverse the direction of the inequality.*

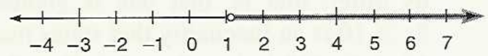
We write the solution sets using interval or set-builder notation and graph the solution sets on the number line using intervals, as shown in section 1–1.

■ ***Example 2–4 A***

Find and graph the solution sets.

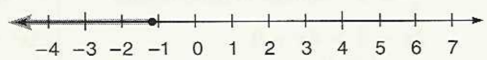
**1.** $3(2 - 4x) > 18 - 22x$

| | |
|---|---|
| $6 - 12x > 18 - 22x$ | Expand the left member |
| $22x - 12x > 18 - 6$ | Add $22x$ and $-6$ to both members |
| $10x > 12$ | Combine like terms |
| $x > \frac{12}{10}$ or $x > \frac{6}{5}$ | Divide both members by 10; reduce |
| $\{x \mid x > 1\frac{1}{5}\}$ | Set-builder notation |
| $(1\frac{1}{5}, \infty)$ | Interval notation |




**2.** $\dfrac{5(3 - 2x)}{2} \geq 12 - x$ A weak inequality is solved in the same way as a strict one

| | |
|---|---|
| $5(3 - 2x) \geq 24 - 2x$ | Multiply both members by 2 |
| $15 - 10x \geq 24 - 2x$ | Multiply in the left member |
| $-8x \geq 9$ | Add $2x$ and $-15$ to both members |
| $x \leq -\frac{9}{8}$ | Divide each member by $-8$; by the multiplication principle we must therefore *reverse the direction of the inequality* |
| $\{x \mid x \leq -1\frac{1}{8}\}$ | Set-builder notation |
| $(-\infty, -1\frac{1}{8}]$ | Interval notation |



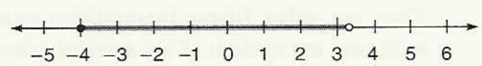

**3.** $-8 < 2 - 3x \leq 14$

This is a **compound inequality** (section 1–1). It means that

$$-8 < 2 - 3x \textit{ and } 2 - 3x \leq 14$$

The compound inequality can be solved in the same way as simple inequalities if we apply the same rule to all three members at a time. *This process is equivalent to solving both the inequalities it represents.*

| | |
|---|---|
| $-10 < -3x \leq 12$ | Add $-2$ to each member |
| $\dfrac{-10}{-3} > \dfrac{-3x}{-3} \geq \dfrac{12}{-3}$ | Divide each member by $-3$; this means we must reverse the direction of the inequalities |
| $3\frac{1}{3} > x \geq -4$ | |
| $-4 \leq x < 3\frac{1}{3}$ | We usually write the lesser quantity on the left in a compound inequality |
| $\left.\begin{array}{l}\{x \mid -4 \leq x < 3\frac{1}{3}\} \\ [-4, 3\frac{1}{3})\end{array}\right\}$ | Solution set |

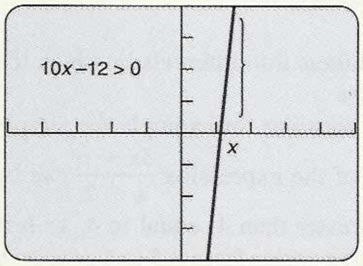



■

Graphical methods can be employed to find approximate solutions to linear inequalities. There are at least two ways to do this. This is illustrated in example 2–4 B, where we do parts 1 and 3 of example 2–4 A one way and part 2 of example 2–4 A a second way.

## ■ Example 2–4 B

Find the solution sets by graphical methods.

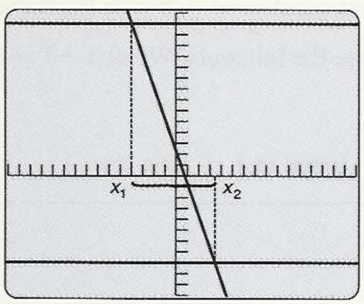



**1.** Solve the inequality $3(2 - 4x) > 18 - 22x$.

We can simplify as
$$\begin{aligned} 3(2 - 4x) &> 18 - 22x \\ 6 - 12x &> 18 - 22x \\ 6 - 12x - 18 + 22x &> 0 \\ 10x - 12 &> 0 \end{aligned}$$

We graph $y = 10x - 12$; the solution is where the graph is greater than zero.

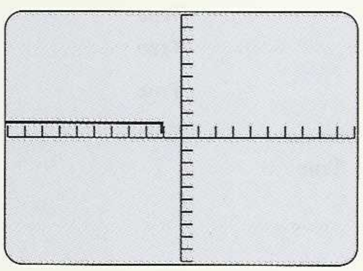
We can see that the graph is greater than zero when $x$ is to the right of where it crosses the $x$-axis. The value $x$ can be found by tracing and zooming to be about 1.2. Thus, $10x - 12 > 0$ when $x > 1.2$, and the solution to the original inequality is $x > 1.2$.




**2.** Solve the inequality $-8 < 2 - 3x \leq 14$ graphically.

Graph the three lines $Y_1 = -8$, $Y_2 = 2 - 3x$, $Y_3 = 14$. See the graph. Use Ymax=15, Ymin=−10, Xmax=15, Xmin=−15.

We can see that the solution is the values of $x$ between $x_1$ and $x_2$, and including $x_1$ (since we did say $2 - 3x \leq 14$). We can use the trace function on the graph of $Y_2$. Do this by selecting [TRACE] and using the up and down arrows to move the cursor to the slanted line, which is the graph of $Y_2$. In this manner we can determine that the value of $x_1$ is about −4 and the value of $x_2$ is about 3.3. Algebraically we determined the solution is exactly: $-4 \leq y < 3\frac{1}{3}$ (example 2–4 A).



**3.** $\dfrac{5(3 - 2x)}{2} \geq 12 - x$ (part 2 of example 2–4 A).

Use the following keystrokes:

[Y =]

5 [(] 3 [−] 2 [X | T] [)] [÷] 2

[TEST] ([2nd] [MATH])

4 (≥)

12 [−] [X | T]

[GRAPH]

The TI-81 is programmed to graph the value 1 where a function is TRUE, and 0 where it is FALSE. Thus the graph indicates the expression $\dfrac{5(3 - 2x)}{2} \geq 12 - x$ is true (1) for $x$ slightly less than −1, and is false (0) everywhere else. Zooming could refine the value, obtaining an answer close to the actual value of $-1\frac{1}{8}$. ■

## Nonlinear inequalities

To solve nonlinear inequalities algebraically we employ a method called the **critical point/test point method.** We can also solve them, approximately, by graphing. This is shown after we examine the algebraic methods.

We first investigate the critical point/test point method.

The **critical points** of an inequality are

1. the solutions to the corresponding equality and
2. the zeros of any denominators.

Critical points are used to divide the real numbers into intervals in which the inequality is either always true or always false.

To get a feeling for why this all works, consider for example the simple nonlinear inequality $\frac{5x+5}{x+2} > 4$. The value of the expression $\frac{5x+5}{x+2}$ can be divided into three categories relative to 4: greater than 4, equal to 4, or less than 4. Look at the following table of values as $x$ varies from $-5$ to 6 as shown in table 2–1. The information shown in table 2–1 is also shown graphically on the number line in figure 2–6. Observe that the number line divides into three intervals. In two of the intervals the statement $\frac{5x+5}{x+2} > 4$ is true, and it is false in one. The points $-2$ and 3 separate the intervals. We call $-2$ and 3 critical points.

| $x$ | $\frac{5x+5}{x+2}$ | Greater than 4 | Equal to 4 | Less than 4 |
|---|---|---|---|---|
| $-5$ | $6\frac{2}{3}$ | True | | |
| $-4$ | $7\frac{1}{2}$ | True | | |
| $-3$ | 10 | True | | |
| $-2$ | undefined | | | |
| $-1$ | 0 | | | True |
| 0 | $2\frac{1}{2}$ | | | True |
| 1 | $3\frac{1}{3}$ | | | True |
| 2 | $3\frac{3}{4}$ | | | True |
| 3 | 4 | | True | |
| 4 | $4\frac{1}{6}$ | True | | |
| 5 | $4\frac{2}{7}$ | True | | |
| 6 | $4\frac{3}{8}$ | True | | |

**Table 2–1**

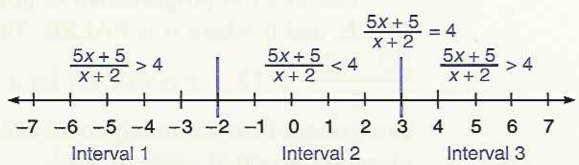



**Figure 2–6**

In an inequality critical points can occur in two places. The first is where the left and right members are equal. The point $x = 3$ is where the left and right members of the inequality $\dfrac{5x+5}{x+2} > 4$ are equal, or in other words, it is a solution to $\dfrac{5x+5}{x+2} = 4$. The second place that critical points can occur is where a denominator is 0. The denominator of $\dfrac{5x+5}{x+2}$ is 0 at $x = -2$, and this is a critical point.

Note that an expression is undefined wherever a denominator is 0, so such a point cannot be part of a solution set.

To locate where $\dfrac{5x+5}{x+2} > 4$ is true we have to check only one value in each interval, since the inequality is true throughout the entire interval or is false throughout the entire interval. We call such a value a test point. Any value in the interval may serve as a test point. According to table 2–1 $\dfrac{5x+5}{x+2} > 4$ is true for $x < -2$ or $x > 3$.

The critical point/test point method for solving nonlinear inequalities is summarized as follows.

**The critical point/test point method for nonlinear inequalities**

Step 1: Find critical points.
- a. Change the inequality to an equality and solve.
- b. Set any denominators involving the variable to zero and solve.

Step 2: Find test points.
- a. Use the critical points found in step 1 to mark intervals on the number line.
- b. Choose one test point from each interval. Any point will do.

Step 3: Locate the intervals which form the solution set.
- a. Try the test points in the original inequality.
- b. Note the intervals where the test point makes the original problem true.

Step 4: Include any of the critical points which make the original inequality true in the solution set.
This will only occur when the original inequality was a weak one.

Step 5: Write the solution set (we will use both set-builder and interval notation).

**Note** Do not attempt to solve a nonlinear inequality such as $\dfrac{5x+5}{x+2} > 4$ by multiplying both members by $x + 2$. We do not know whether $x + 2$ is positive or negative, so we do not know whether we should reverse the $>$ or not.

■ ***Example 2–4 C*** Solve and graph the solutions to the following nonlinear inequalities.

**1.** Solve the nonlinear inequality $\dfrac{x+3}{x-4} \geq 5$.

**Step 1:** Find the critical points

**a.** $\dfrac{x+3}{x-4} = 5$ — Change $\geq$ to $=$ and solve

$x + 3 = 5(x - 4)$ — Multiply both sides by $x - 4$

$x + 3 = 5x - 20$

$23 = 4x$

$5\frac{3}{4} = x$ — Solve for $x$

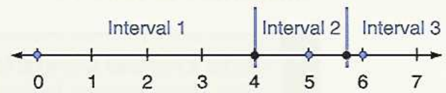
**b.** $x - 4 = 0$ — Set the denominator $x - 4$ equal to zero

$x = 4$ — Solve for $x$

Critical points: 4, $5\frac{3}{4}$ — At $5\frac{3}{4}$ we also have equality. Since this is a weak inequality, $5\frac{3}{4}$ is part of the solution set

**Step 2:** Find test points.

Plot the points 4 and $5\frac{3}{4}$ from step 1. These form the three intervals shown in the figure. Choose one point from each interval to be a test point. We choose 0, 5, and 6.




**Step 3:** Test the original inequality using these test points and note where the original inequality is true.

The original inequality is $\dfrac{x+3}{x-4} \geq 5$. We test it for each value of $x$, 0, 5, and 6.

| | **Interval 1** | **Interval 2** | **Interval 3** |
|---|---|---|---|
| **Test point** | 0 | 5 | 6 |
| | $\dfrac{0+3}{0-4} \geq 5$ | $\dfrac{5+3}{5-4} \geq 5$ | $\dfrac{6+3}{6-4} \geq 5$ |
| | $-\frac{3}{4} \geq 5$ | $\dfrac{8}{1} \geq 5$ | $\dfrac{9}{2} \geq 5$ |
| | False | True | False |

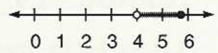
The original inequality is true in interval 2.

**Step 4:** From step 1, the inequality is satisfied at $5\frac{3}{4}$, so this is part of the solution set.

**Step 5:** The solution is those intervals that had a test point that made the original inequality true, as well as any critical points for which the

original inequality is true. In this case the solution is interval 2 together with the critical point $5\frac{3}{4}$. This result can be graphed as shown in the figure and described algebraically as

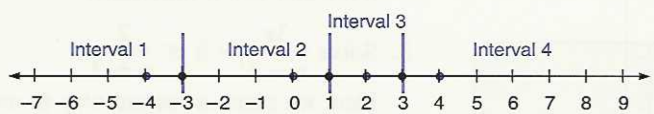
$\{x \mid 4 < x \leq 5\tfrac{3}{4}\}$ Set-builder notation
$(4,5\tfrac{3}{4}]$ Interval notation




**2.** Solve the nonlinear inequality $x^3 - x^2 - 9x + 9 \geq 0$.
We solve by using the five steps illustrated above.

$x^3 - x^2 - 9x + 9 = 0$ Change $\geq$ to $=$ and solve
$x^2(x - 1) - 9(x - 1) = 0$ Factor by grouping
$(x - 1)(x^2 - 9) = 0$
$(x - 1)(x - 3)(x + 3) = 0$ Set each factor to 0
$x - 1 = 0$ or $x - 3 = 0$ or $x + 3 = 0$
$x = 1$ or $x = 3$ or $x = -3$

Thus, our critical points are $-3$, 1, and 3. These points are also part of the solution set since they are solutions to the original weak inequality. These form the four intervals shown in the figure. Choose $-4$, 0, 2, and 4 for test points.



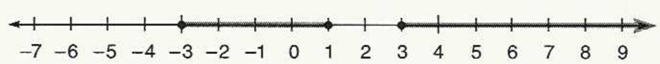

The original inequality is $x^3 - x^2 - 9x + 9 \geq 0$. We compute with the factored form of the left expression $(x - 1)(x - 3)(x + 3)$, since these computations are easier and give the same result.

| | **Interval 1** | **Interval 2** | **Interval 3** | **Interval 4** |
|---|---|---|---|---|
| **Test point** | $-4$ | 0 | 2 | 4 |
| | $(-5)(-7)(-1) \geq 0$ | $(-1)(-3)(3) \geq 0$ | $1(-1)(5) \geq 0$ | $3(1)(7) \geq 0$ |
| | $-35 \geq 0$ | $9 \geq 5$ | $-5 \geq 0$ | $21 \geq 0$ |
| | False | True | False | True |

The solution is intervals 2 and 4, together with the critical points. This is written as

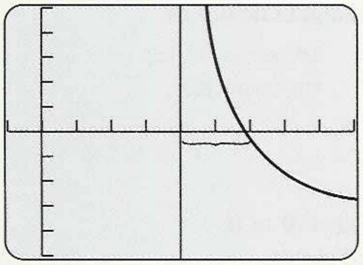
$\{x \mid -3 \leq x \leq 1 \text{ or } x \geq 3\}$ Set-builder notation
$[-3,1]$ or $[3,\infty)$ Interval notation

and is graphed as shown in the figure.




Example 2–4 D shows how to solve nonlinear inequalities by graphing.

■ ***Example 2–4 D*** Solve the following nonlinear inequalities.

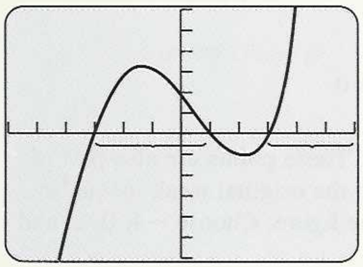


**1.** $\dfrac{x+3}{x-4} \geq 5$ (This is example 2–4 C, part 1.)

$\boxed{\text{Y=}}$ $\boxed{(}$ $\boxed{\text{X|T}}$ $\boxed{+}$ 3 $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$ $\boxed{\text{X|T}}$ $\boxed{-}$ 4 $\boxed{)}$ $\boxed{-}$ 5

$\boxed{\text{RANGE } -1,9,-5,5}$

It is easier to graph the equivalent expression $\dfrac{x+3}{x-4} - 5 \geq 0$. By suitably using trace and zoom we can see that this is greater than or equal to zero above 4 and less than or equal to 5.7, approximately, or $4 < x \leq 5.7$. (Note that the expression is not defined at $x = 4$.) In example 2–4 C we found the exact answer to be $4 < x \leq 5\frac{3}{4}$.



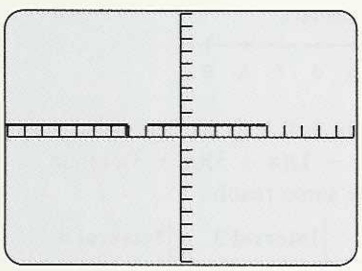
**2.** $x^3 - x^2 - 9x + 9 \geq 0$ (part 2 from example 2–4 C).
Graph $y = x^3 - x^2 - 9x + 9 \geq 0$.

$\boxed{\text{Y=}}$ $\boxed{\text{X|T}}$ $\boxed{\text{MATH}}$ 3 $\boxed{-}$ $\boxed{\text{X|T}}$ $\boxed{x^2}$ $\boxed{-}$ 9 $\boxed{\text{X|T}}$ $\boxed{+}$ 9

$\boxed{\text{RANGE } -6,6,-30,30}$

YSCL = 5.

The values of $x$ where the graph is greater than or equal to zero can be seen to be $-3 \leq x \leq 1$ or $x \geq 3$. This can be confirmed by using the trace and zoom functions.



**3.** Solve $\dfrac{3x}{x-5} - 3 \leq \dfrac{2}{x+2}$.

Here we show a second way to use the graphing calculator to obtain solutions to inequalities. As illustrated in section 2–3 we can use the capability of the calculator to graph the value one where an expression is true, and zero where it is false.

$\boxed{\text{Y=}}$ $\boxed{(}$ 3 $\boxed{\text{X|T}}$ $\boxed{\div}$ $\boxed{(}$ $\boxed{\text{X|T}}$ $\boxed{-}$ 5 $\boxed{)}$ $\boxed{)}$ $\boxed{-}$ 3

$\boxed{\text{TEST}}$ 6 2 $\boxed{\div}$ $\boxed{(}$ $\boxed{\text{X|T}}$ $\boxed{+}$ 2 $\boxed{)}$ $\boxed{\text{RANGE } -10,10,-10,10}$

Zooming and tracing could be used to see that the values are $-2$ and 5, and the third value is close to $-3\frac{1}{13}$, or $\approx -3.077$. Algebraic solution shows that the solution is $(-\infty,-3\frac{1}{13})$ or $(-2,5)$. ■

Several examples of applications of linear and nonlinear inequalities are illustrated in example 2–4 E.

## ■ Example 2–4 E

Applications of inequalities.

1. Find the domain of the expression $\sqrt{2x - 3}$.
   We require that the expression under a square root radical be nonnegative to obtain a real value. Thus,
$$2x - 3 \geq 0$$
$$2x \geq 3$$
$$x \geq \tfrac{3}{2}$$
   The domain is $\{x \mid x \geq \frac{3}{2}\}$.

2. To pass a company's aptitude test a candidate must average 80 or above on three tests. If a certain candidate has had scores of 72 and 81 on the first two tests, what is the minimum score that must be obtained on the third test to have a passing average? If $T$ represents the test score on the third test, we require that $\dfrac{72 + 81 + T}{3} \geq 80$.
$$\frac{72 + 81 + T}{3} \geq 80$$
$$72 + 81 + T \geq 240 \quad \text{Multiply each member by 3}$$
$$T \geq 87 \quad \text{Subtract } (72 + 81) \text{ from both members}$$
   Thus the candidate must score 87 or above on the third test. ■

### Mastery points

**Can you**

- Solve linear inequalities in one variable using the algebraic methods presented?
- Solve nonlinear inequalities in one variable using the critical point/test point method?

## Exercise 2–4

Find and graph the solution sets to the following linear inequalities.

**1.** $5x + 2 > 3x - 8$
**2.** $12x - 3 < 6x + 6$
**3.** $3x - 7 \leq x - 4$
**4.** $9x + 2 \geq 3 - 2x$
**5.** $2(3x - 3) \geq 9x + 1$
**6.** $5 - 3(x - 1) < 6x$
**7.** $9x - 4(2 - x) < 20$
**8.** $16 - 3(2x - 3) \geq 4x + 6$
**9.** $-3(x - 2) + 2(x + 1) \geq 5(x - 3)$
**10.** $2x + 17 < -4(2x - 5) - x + 7$
**11.** $x < 27 - 9x$
**12.** $0 \geq 9x - 36$
**13.** $5x + 18 \geq 0$
**14.** $4(2 - 3x) + 3 \geq 20$
**15.** $6x - 12 \leq 3x - 2(x + 6)$
**16.** $3x - 5 \leq 5x + 2$
**17.** $9x \geq 4x + 3$
**18.** $3(3x - 3) \geq 11$
**19.** $5x - 3(x - 1) \geq 6x$
**20.** $9 - 3(1 - x) < 20x$
**21.** $12 + 3(3 - 2x) > 4(x + 6)$
**22.** $10(x - 2) - 2(x + 1) \geq 5(x - 3)$
**23.** $17 \leq 3(3x - 5) - (x + 7)$
**24.** $3(x + 2) < 27 - 9x$
**25.** $6 - x > 9x - 36$
**26.** $12 - 6x \leq 3x - 2(x + 5)$

Find and graph the solution set to the following nonlinear inequalities.

**27.** $x^2 + 5x \le 24$ **28.** $\dfrac{x-2}{x+1} \le 0$ **29.** $(x-3)(x+1)(x-1) \le 0$

**30.** $x^2 - 5x - 24 > 0$ **31.** $q(3q-5) < 0$ **32.** $2m^2 - 3m \ge 0$

**33.** $r(2r-3)(r+1) \le 0$ **34.** $3w^2 + 5 \ge 16w$ **35.** $(x^2-4)(x-1) < 0$

**36.** $(x-9)(x^2-9) \ge 0$ **37.** $(x^2-4x+4)(x^2-4) \le 0$ **38.** $(x-1)^2(x+2)^3 > 0$

**39.** $\dfrac{(x+1)^2}{2x-3} \ge 0$ **40.** $\dfrac{3-x}{3+x} - 4 < 0$ **41.** $\dfrac{3}{x} > 0$ **42.** $\dfrac{4}{x-2} < 0$

**43.** $\dfrac{x^2-1}{x-3} \le 0$ **44.** $\dfrac{x+3}{x^2-4} \ge 0$ **45.** $\dfrac{x^2-10x+25}{x^2-x-6} \le 0$ **46.** $\dfrac{2x}{x+1} \ge 3$

**47.** $\dfrac{x-3}{x+5} + x \ge 3$ **48.** $\dfrac{2p-1}{2p} \le 4$ **49.** $\dfrac{x}{x+1} - \dfrac{2}{x+3} \le 1$ **50.** $\dfrac{x+2}{x-3} \ge \dfrac{x+4}{x-6}$

Find the domain of the following expressions.

**51.** $\sqrt{4x-10}$ **52.** $\sqrt{3x+1}$ **53.** $\sqrt{9-2x}$ **54.** $\sqrt{6-\dfrac{x}{2}}$

**55.** $\sqrt{x^2-5x-6}$ **56.** $\sqrt{x^2+x-30}$ **57.** $\sqrt{4x^2-4x-3}$ **58.** $\sqrt{3x^2+2x-5}$

**59.** The final grade in a certain course is the average of four exams. Each exam is scored from 0 to 100 points. A student has received grades of 66, 71, and 84 on the first three exams. The student would like to achieve a final average grade of at least 75.

**a.** What is the minimum score that this student must have on the fourth exam?

**b.** What is the highest average that this student can achieve for the course?

**60.** A student in the course described in problem 59 has received a grade of 60 on the first test.

**a.** If the student would like to have an average of 75 or above at the end of the course what must be the average of the remaining three exams?

**b.** What is the highest average which this student can achieve for the course?

**61.** The perimeter of a certain rectangle must be less than 100 feet. The length of this rectangle is 30 feet. Find all values of the width that would meet these conditions. The width must be a positive number.

**62.** The perimeter of a certain rectangle must be between 50 feet and 200 feet. The length of this rectangle is 20 feet. Find the range of values that the width must be to meet these conditions.

**63.** The perimeter of a square must be greater than 16 inches but less than 84 inches. Find all values of the length of a side that will meet these conditions. Note that, since all four sides of a square have the same length, the perimeter $P$ of a square is $4s$, where $s$ is the length of a side.

**64.** In a course there are four in-class tests and a final exam. The exam counts 30% of the grade, and the tests are all counted equally. If $T$ = test average, $E$ = exam, and $G$ = final grade, then a relation that expresses this would be $0.7T + 0.3E = G$. A certain student has an average of 78 for the four tests, and would like to get a final average of 80 or above. Write an inequality that describes the final exam grade $E$ necessary for this result.

**65.** An electronics circuit has two resistances in parallel. The value of one resistance is $x$ (ohms) and the second resistance is 10 ohms greater than this. The total resistance must not exceed 40 ohms. Under these circumstances the relation $\dfrac{1}{x} + \dfrac{1}{x+10} \ge \dfrac{1}{40}$ will be true. Solve for $x$.

**66.** In a certain rectangle the width is 7 less than the length. The area must exceed 30 square units. Find the restrictions on the length that will give this result.

**67.** A certain printing press can print 1,500 newspapers in a certain time $t$ (in minutes). A second press is always three minutes faster, so it takes $t - 3$ minutes to do the same thing. Running together it is necessary for the presses to produce 3,000 papers per minute. Under these circumstances the relation $\dfrac{1{,}500}{t} + \dfrac{1{,}500}{t-3} \ge 3{,}000$ holds. Solve this inequality, and then note that $t > 0$ and $t - 3 > 0$ must also be true to find a range of values for $t$ that make sense.

**68.** The zoning bylaws of a certain town (Carlisle, Mass.) require, among other things, that building lots have a ratio of area $A$ to perimeter $P$ conforming to the relation $\frac{16A}{P^2} > 0.4$.

**a.** Solve this relation for $A$.
**b.** Solve this relation for $P$.

**69.** Determine which of the following building lots conform to the zoning bylaw of the previous problem.

| | Length | Width | | Length | Width |
|---|---|---|---|---|---|
| **a.** | 100′ | 50′ | **b.** | 200′ | 50′ |
| **c.** | 300′ | 50′ | **d.** | 400′ | 50′ |
| **e.** | 500′ | 50′ | **f.** | 600′ | 50′ |

**70.** Consider how the relation stated in problem 68 applies to rectangular lots. If $L$ is length and $W$ is width for a rectangular lot, then

$$A = LW \text{ and } P = 2(L + W).$$

Let $k$ be the ratio of length to width $\left(\text{i.e., } k = \frac{L}{W}\right)$. Show that $k$ must fall in the interval $4 - \sqrt{15} < k < 4 + \sqrt{15}$.

**71.** A machine can produce 30 bolts per hour when the cutting tool is new. Then for the first 10 hours of use, it produces two fewer bolts per hour than the previous hour. Under these conditions the total number of bolts produced after $x$ hours is given by the expression $30x - x^2$, $0 \le x \le 10$. To find out how many hours are required for the total production to be 100 bolts or more we solve $30x - x^2 \ge 100$. Solve this inequality and find $x$ to the nearest minute.

**72.** The critical point/test point method can be used to solve simple linear inequalities. Use it to solve $3x - 5 \le 6x$.

## *Skill and review*

**1.** Solve $\frac{2x - 3}{4} = 2$.

**2.** Solve $\frac{2x^2 - 4}{7} = x$.

**3.** If $|x| = 8$, then
**a.** $x = 8$ **b.** $x = -8$ **c.** $x = 8$ or $x = -8$
**d.** $-8 < x < 8$

**4.** If $|x| < 8$, then
**a.** $x = 0$ **b.** $-8 < x < 8$ **c.** $x < -8$ or $x > 8$
**d.** $x < 7$

**5.** If $|x| > 8$ then
**a.** $x = 0$ **b.** $-8 < x < 8$ **c.** $x < -8$ or $x > 8$
**d.** $x < 7$

**6.** Is the value 3 a solution to $\left|\frac{1 - x}{2}\right| < x$?

**7.** For what value(s) of $x$ is $|x| = |-x|$ true?

# *2–5 Equations and inequalities with absolute value*

A computer program is used to buy and sell stock. A certain stock is selling for $32\frac{1}{8}$ points ($32.125 per share). The computer is set to alert someone if the price of the stock changes by more than $1\frac{1}{8}$ points. Thus the prices $x$ that would set off this alert are described by $|32\frac{1}{8} - x| \ge 1\frac{1}{8}$.

As this problem illustrates, absolute values and inequalities can be used to describe certain applied situations. The combination is most useful where the difference between two quantities must meet some restriction. We can often use the properties described in this section to find the solution set when one or both members of an equation or inequality is the absolute value of an expression involving a variable.

## Equations with absolute value

What would $|x| = 4$ mean about $x$? It can be seen that $x$ must be either 4 or $-4$. That is, if $|x| = 4$ then $x = 4$ or $x = -4$. We can generalize this into the following principle.

> **Equations with absolute value**
> [1] If $|x| = b$ and $b \geq 0$, then $x = b$ or $x = -b$.

■ ***Example 2–5 A***

Solve the equation $|4x - 2| = 8$.

$$|4x - 2| = 8$$

Rewrite using property [1] then solve each equation

$$4x - 2 = 8 \quad \text{or} \quad 4x - 2 = -8$$
$$4x = 10 \qquad\qquad 4x = -6$$
$$x = \tfrac{5}{2} \qquad\qquad x = -\tfrac{3}{2}$$

$\{-\frac{3}{2}, \frac{5}{2}\}$ Solution set ■

## Inequalities with absolute value

The following two properties describe many situations involving absolute values and inequalities. These can be thought of as rewriting rules, just as property [1] is. These properties allow us to rewrite a statement that involves absolute values as equivalent sets of statements that do not involve absolute value.

> **Inequalities with absolute value**
> [2] If $|x| < b$ and $b > 0$, then $-b < x < b$.
> [3] If $|x| > b$ and $b \geq 0$, then $x > b$ or $x < -b$.

Figure 2–7 illustrates these rewriting rules.

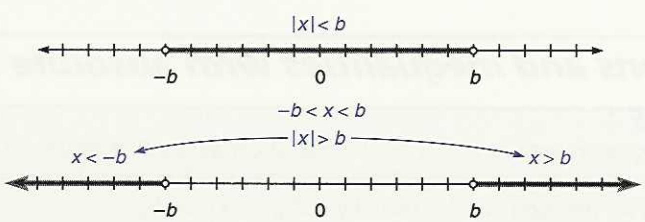



**Figure 2–7**

**Note** 1. For property [2], if $b < 0$, then the solution set is the empty set.
2. For property [3], if $b < 0$ then any value of $x$ is a solution, since $|x| \geq 0 > b$, if $b$ is negative.

Similar properties apply for weak inequalities.

Although properties [2] and [3] are always true, they are most useful when the expression represented by $x$ is linear. These cases are illustrated in example 2–5 B.

■ *Example 2–5 B*

Solve. State the solution in set-builder notation and graph the solution set.

**1.** $\left|\dfrac{3x-1}{4}\right| > 3$

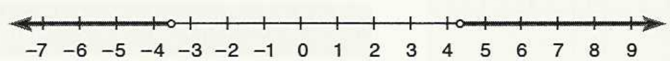
$\dfrac{3x-1}{4} > 3$ or $\dfrac{3x-1}{4} < -3$ — Rewrite using property [3]

$3x - 1 > 12$ $\quad$ $3x - 1 < -12$ — Multiply each member by 4

$3x > 13$ $\quad$ $3x < -11$

$x > 4\frac{1}{3}$ $\quad$ $x < -3\frac{2}{3}$ — Solve the linear inequality

$\{x \mid x < -3\frac{2}{3} \text{ or } x > 4\frac{1}{3}\}$ — Solution set, set-builder notation



**2.** $|5 - 2x| \leq 8$

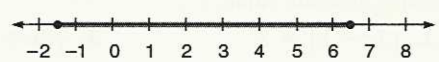
$-8 \leq 5 - 2x \leq 8$ — Rewrite using property [2]

$-13 \leq -2x \leq 3$ — Add $-5$ to each of the three members

$\frac{13}{2} \geq x \geq -\frac{3}{2}$ — Divide each member by $-2$

$-\frac{3}{2} \leq x \leq \frac{13}{2}$ — It is customary to put the smallest member on the left

$\{x \mid -1\frac{1}{2} \leq x \leq 6\frac{1}{2}\}$ — Solution set



**3.** $|x - 1| > -1$

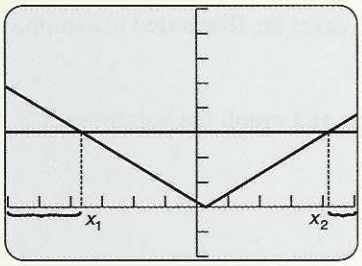
Since $|x - 1| \geq 0$ for any value of $x$, and since $0 > -1$, we can conclude that $|x - 1| > -1$ is true for any value of $x$. Thus, the solution set is $R$. The graph is the entire number line. ■

Example 2–5 C shows how to find approximate solutions to these problems using the graphing calculator.

■ *Example 2–5 C*

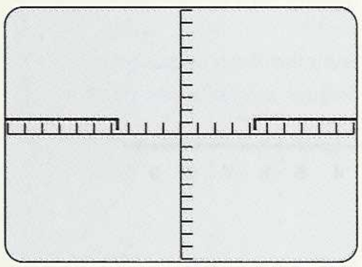
Solve $\left|\dfrac{3x-1}{4}\right| > 3$ (part 1 of example 2–4 B).

As illustrated in the last two sections, there are two ways to solve this problem graphically.




**Method 1:** Graph the equations $Y_1 = \left|\dfrac{3x - 1}{4}\right|$ and $Y_2 = 3$. The graph is shown. The solution is where $Y_1 > Y_2$. This is to the right of $x_2$ and the left of $x_1$. We can use trace and zoom to estimate the values as 4.3 and $-3.7$. Thus the approximate solution is $x > 4.3$ or $x < -3.7$.

$\boxed{\text{Y=}}$ $\boxed{\text{ABS}}$ $\boxed{(}$ $\boxed{(}$ 3 $\boxed{\text{X|T}}$ $\boxed{-}$ 1 $\boxed{)}$ $\boxed{\div}$ 4 $\boxed{)}$ $\boxed{\text{ENTER}}$ 3 $\boxed{\text{RANGE } -6,6,-2,8}$



**Method 2:** Graph $Y_1$=abs((3X − 1)/4) > 3. (Remember, > is $\boxed{\text{2nd}}$ $\boxed{\text{MATH}}$ 3.)

The second graph is shown for standard settings ( $\boxed{\text{ZOOM}}$ 6).

## Mastery points

**Can you**
- Solve equations involving absolute value?
- Solve inequalities involving absolute value?

## Exercise 2–5

Solve the following equations involving absolute value.

**1.** $|5x| = 8$ **2.** $|3x - 1| = 3$ **3.** $|2x + 5| = 6$ **4.** $|x + \frac{3}{5}| = 1$

**5.** $|3 - 2x| = 5$ **6.** $|9x + 1| = 7$ **7.** $|2x + 8| = 0$ **8.** $\left|\dfrac{3 - 2x}{4}\right| = 0$

$\boxed{9.}$ $\left|\dfrac{3x - 5}{4}\right| = 1$ $\boxed{10.}$ $|x^2 - 2x| = 3$ **11.** $|2x^2 - 5x| = 9$ **12.** $\left|\dfrac{x^2 + 6x}{2}\right| = 8$

Solve the following inequalities involving absolute value. State the solution in set-builder notation and graph the solution set.

**13.** $|3 + 6x| > 4$ **14.** $|4 - 3x| \le \frac{1}{2}$ $\boxed{15.}$ $\left|\dfrac{x - 4}{3}\right| \le 10$ **16.** $\left|\dfrac{4x - 1}{5}\right| < 1$

**17.** $|2 - 5x| < -3$ **18.** $|2 - 5x| > -3$ **19.** $|-5(2x + 3)| \ge 8$ **20.** $\left|\dfrac{2x}{3} - x\right| \le 4$

$\boxed{21.}$ $\left|3x - \dfrac{x}{3}\right| > 3$ **22.** $|-x + 11| < \frac{3}{8}$ **23.** $|x + 5| > -2$ **24.** $|3x - \frac{1}{5}| < 0$

Solve the following equations and inequalities involving absolute value. State the solution in set-builder notation.

**25.** $|3x| > 22$ **26.** $|4x + 3| > 1$ $\boxed{27.}$ $25 < |5 - 2x|$ **28.** $17 < |4x - 1|$

**29.** $\left|\dfrac{x - 2}{4}\right| > 9$ **30.** $\left|\dfrac{3 - 2x}{5}\right| > 8$ **31.** $|3x| = 22$ **32.** $|4x + 3| = 1$

**33.** $|5 - 2x| = 25$ **34.** $17 = |4x - 1|$ **35.** $\left|\dfrac{x - 2}{4}\right| = 9$ **36.** $8 = \left|\dfrac{3 - 2x}{5}\right|$

**37.** $|3x| < 22$ **38.** $|4x + 3| < 1$ **39.** $|5 - 2x| < 25$ **40.** $17 > |4x - 1|$

**41.** $\left|\dfrac{x-2}{4}\right| < 9$

**42.** $8 > \left|\dfrac{3-2x}{5}\right|$

**43.** $|4x| = 5$

**44.** $|3x+1| < 4$

**45.** $5 > |6x-3|$

**46.** $|9x-17| > 5$

**47.** $|3x| \geq 8$

**48.** $6 \leq |9x+1|$

**49.** $\left|\dfrac{2x-3}{4}\right| \leq 17$

**50.** $\dfrac{1}{2} < \left|\dfrac{2x+5}{9}\right|$

**51.** $\left|\dfrac{4x+7}{8}\right| \geq \dfrac{4}{9}$

**52.** $16 \geq \left|\dfrac{3x-1}{5}\right|$

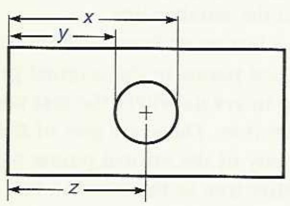
**53.** $2 < |-x|$

**54.** $4 > |11-6x|$

**55.** $2 \leq \left|\dfrac{x+2}{3}\right|$

**56.** $\dfrac{3}{4} \leq \left|\dfrac{3x+1}{8}\right|$

**57.** In the diagram the dimension $z$ is $6\frac{1}{2}$ inches. If the circle has radius $\frac{7}{8}$ inches, then the dimensions $x$ and $y$ are the two solutions to the equation $|6\frac{1}{2} - w| = \frac{7}{8}$. Solve this for $w$, and thereby find the values of $x$ and $y$.




**58.** A computer program is used to buy and sell stock. A certain stock is selling for $32\frac{1}{8}$ points (\$32.125 per share). The computer is set to alert someone if the price of the stock changes by more than $1\frac{1}{8}$ points. Thus the prices $x$ that would set off this alert are described by $|32\frac{1}{8} - x| \geq 1\frac{1}{8}$. Solve this inequality.

**59.** The total number of bolts produced after $x$ hours by a certain machine is given by the expression $x^2 - \dfrac{x}{3}$. A different machine produces $x^2$ bolts after $x$ hours. A production engineer is interested in knowing when the two machines have produced the same number of bolts, plus or minus 6 bolts. This can be determined by solving $\left|\left(x^2 - \dfrac{x}{3}\right) - (x^2)\right| \leq 6$. Solve this expression for $x$. Assume $x \geq 0$.

**60.** Beginning with $(x-y)^2 \geq 0$, show that
$$\frac{x^2+y^2}{2} \geq xy.$$

## *Skill and review*

1. If $x = 1$ and $y = -3$ is the statement $2x - y = 5$ true?
2. If $x = -3$ and $y = -11$ is the statement $2x - y = 5$ true?
3. If $x = 3$ and $y = 1$ is the statement $2x - y = 5$ true?
4. If $x = 0$ and $y = -5$ is the statement $2x - y = 5$ true?
5. If $x = 2$ and $y = -3$, which of the statements is true?
   **a.** $3x + y = 3$ **b.** $-x + 5y = -17$ **c.** $y + 9 = 3x$ **d.** $x = y + 5$
6. If $x = -2$ and $y = 4$, which of the statements is true?
   **a.** $3x + y = -2$ **b.** $-x + 5y = 18$ **c.** $y + 10 = 3x$ **d.** $x = y + 6$
7. Solve $2x + y = 8$ for $y$.
8. Solve $x - 2y = 4$ for $y$.

## *Chapter 2 summary*

- **To solve a linear equation** Form a sequence of equivalent equations until one is sufficiently simple to solve by inspecting it. Form these equations by using the following steps:
  1. Clear any denominators by multiplying each term by the least common denominator of all the terms.
  2. Perform indicated multiplications (remove the parentheses).
  3. Use the addition property of equality so that all terms with the variable are in one member of the equation, and all other terms are in the other member.
  4. If necessary factor out the variable from the terms containing it.
  5. Divide both members of the equation by the coefficient of the variable.

- **Quadratic equation in one variable** An equation which can be put in the standard form $ax^2 + bx + c = 0$, $a,b,c \in R$, $a \neq 0$.
- **The quadratic formula and the factors of a quadratic expression** If $ax^2 + bx + c = 0$ and $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and $ax^2 + bx + c$
$= a\left(x - \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(x - \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$.
- **Solving quadratic equations**
  Put the equation in standard form.
  If the quadratic expression can be factored use the zero product property.
  When $b$ in $ax^2 + bx + c = 0$ is zero use the method called extracting the roots.
  When the methods mentioned above do not apply use the quadratic formula.
- **Pythagorean theorem** In any right triangle where $c$ is the longest side (the hypotenuse, always opposite the right angle), and $a$ and $b$ are the remaining two sides, $a^2 + b^2 = c^2$.
- **Solving radical equations** Raise each member of the equation to the appropriate power. We may get solutions that are not solutions to the original equation.
- **Addition property of inequality** For any algebraic expressions $A$, $B$, and $C$, if $A < B$ then $A + C < B + C$.
- **Multiplication property of inequality** For any algebraic expressions $A$, $B$, and $C$,
  1. if $C > 0$ and $A < B$ then $AC < BC$,
  2. if $C < 0$ and $A < B$ then $AC > BC$.
- **Solving linear inequalities** Solve linear inequalities exactly the same way as linear equations except that if we multiply (or divide) both members of an inequality by a negative value we must reverse the direction of the inequality.
- **Solving nonlinear inequalities** To solve nonlinear inequalities employ a method called the critical point/test point method:
  1. Solve the corresponding equality.
  2. Solve for any zeros of any denominators. Steps 1 and 2 produce critical points.
  3. Use the critical points found in steps 1 and 2 to mark intervals on the number line.
  4. Choose a test point from each interval.
  5. Try the test points in the original problem.
  6. Note the intervals where the test point makes the original problem true. These are part of the solution set.
  7. Include any of the critical points which make the original inequality true in the solution set.
  8. Write the solution set using set-builder notation.
- **Solving equations involving absolute value** Use the property
  [1] If $|x| = b$ and $b \geq 0$, then $x = b$ or $x = -b$.
- **Solving absolute value and inequality problems** Use the properties
  [2] If $|x| < b$ and $b > 0$, then $-b < x < b$.
  [3] If $|x| > b$ and $b \geq 0$, then $x > b$ or $x < -b$.

## Chapter 2 review

**[2–1]** Solve the following linear equations by specifying the solution set.

1. $\frac{3}{5}x - 4 = 2 - \frac{3}{4}x$
2. $\frac{3}{4}(5x - 3) - 3x + 19 = 0$
3. $-2[\frac{1}{2} - 2(5 - x) + 2] - \frac{3}{2}x = 0$
4. $4(x - 2) = -(7 - 4x) - 1$
5. $x - \frac{3}{8}x = \frac{1}{4}x$

Find approximate answers to the following problems. Round the answer to 4 digits of accuracy.

6. $13.5x - 22.3x = 0.03(1{,}200 - 2{,}113x)$
7. $11.4 - 3.5x - \sqrt{2}[9.2 - 1.5(\frac{3}{8}x - 5.3) - x] = 0$

Solve the literal equations in problems 8 through 12 for the variable indicated.

8. $m = -p(Q - x)$; for $Q$
9. $R = W - k(2c + b)$; for $b$
10. $P = \dfrac{n}{5}(P_2 - P_1) - c$; for $P_1$
11. $\dfrac{x + 2y}{3 - 2y} = x$; for $x$
12. $\dfrac{x + y}{x} = y$; for $x$
13. A total of \$8,000 is invested, part at 7% and part at 5%. The income from these investments for one year is \$471. How much was invested at each rate?

**14.** A total of \$15,000 was invested; part of the investment made a 12% gain, but the rest had a 10% loss. The net *loss* from the investments was \$720. How much was invested at each rate?

**15.** A total of \$5,000 was invested, part at 5% and part at 9%. If the income for one year from the 9% investment was \$44 more than the income from the 5% investment, how much was invested at each rate?

**16.** A company has a fertilizer that is 10% phosphorous, and a second that is 25% phosphorous. How much of each must be mixed to obtain 2,000 pounds of a mixture that is 15% phosphorous?

**17.** A company has 15 tons of material that is 40% copper. How much material that is 55% copper must be mixed with this to obtain a material that is 45% copper?

**18.** One computer printer can print out 12,000 labels in 35 minutes; a second printer takes 1 hour 10 minutes to do the same job. How long would it take the printers, running at the same time, to print out a total of 12,000 labels?

**19.** A boat moves at 12 mph in still water. If the boat travels 30 miles downstream in the same time it takes to travel 18 miles upstream, what is the speed of the current?

**20.** A boat travels 50 kilometers upstream in the same time that it takes to travel 80 kilometers downstream. If the stream is flowing at 5 km per hour, what is the speed of the boat in still water?

**[2–2]** Solve the following quadratic equations for $x$ by factoring.

**21.** $2x^2 - 7x - 30 = 0$ **22.** $x = \dfrac{5}{2} + \dfrac{3}{2x}$

**23.** $(2x - 5)^2 = 40 - 16x$ **24.** $6a^2x^2 + 7abx = 5b^2$

Solve the following equations by extracting the roots.

**25.** $27x^2 - 40 = 0$ **26.** $(2x - 3)^2 = 12$

**27.** $4(x + 1)^2 = 8$

**28.** $a(bx + c)^2 = d$; assume $a, d > 0$

Solve the following quadratic equations using the quadratic formula.

**29.** $5y^2 - 15y - 6 = 0$ **30.** $2y^2 - \dfrac{2y}{3} + 1 = 0$

**31.** $(z + 4)(z - 1) = 5z + 4$ **32.** $\dfrac{1}{x + 2} - 6x = 25$

Use the quadratic formula to help factor the following quadratic equations.

**33.** $3x^2 - 8x - 12$ **34.** $5x^2 + 6x - 4$

**35.** In a right triangle the length of the hypotenuse is 26, and one side is 14 units longer than the other. Find the length of the longer side.

**36.** A rectangle that is $8\frac{1}{2}$ centimeters long and 4 centimeters wide has its dimensions increased by adding the same amount. The area of this new rectangle is 90 cm$^2$. What are the dimensions of the new rectangle?

**37.** A manufacturer finds that the total cost $C$ for a product is expressed by $C = 250x^2 - 24{,}000$, and the total revenue $R$ at a price of \$200 per unit to be $R = 200x - 1{,}000$, where $x$ is the number of units sold. What is the break even point (where total cost = total revenue) to the nearest unit?

**38.** Two pipes can be used to empty the water behind a dam. When both pipes run together it takes 40 hours to empty the dam. When they are used separately it takes the smaller pipe 12 hours longer to empty the dam than the larger pipe. Find the time required for the larger pipe to empty the dam alone, to the nearest hour.

**39.** When flying directly into a 25 mph wind it takes an aircraft 1 hour longer to travel 300 miles than when there is no wind. Find the time required for the aircraft to fly this distance in no wind.

**40.** Determine the domain of the expression $\dfrac{3x}{x - 3}$.

**41.** Determine the domain of the expression $\dfrac{2x - 5}{x^2 + 8x + 12}$.

Solve the following quadratic equations. You may want to use substitution.

**42.** $2x^4 - 27x^2 + 81 = 0$

**43.** $(x - 3)^2 - 8(x - 3) - 20 = 0$

**44.** $2t + 7\sqrt{t} - 15 = 0$ **45.** $x^{3/2} - 9x^{3/4} + 8 = 0$

**46.** $y^{-4} - 32 = 4y^{-2}$

**[2–3]** Find the solution set for each equation.

**47.** $\sqrt{x(6x + 5)} = 1$ **48.** $\frac{2}{3}\sqrt{9z - 1} - \sqrt{5z} = 0$

**49.** $\sqrt{5w + 1} = 5w - 19$

**50.** $\sqrt{3n + 1} - \sqrt{n + 1} = 2$

**51.** $\sqrt[4]{2y - 3} = 2$

Solve the following formulas for the indicated variable.

**52.** $\sqrt{(2x-1)(2k+1)} = x + k$; for $x$

**53.** $r = \sqrt{\dfrac{A}{\pi} - AR^2}$; for $A$

**[2–4]** Find and graph the solution sets to the following linear inequalities.

**54.** $-9x - 4(2x - 3) < 0$

**55.** $3(x - 1) + 2(3 - 2x) \geq 5(x - 3)$

**56.** $2x - 12 \leq 3x - \frac{1}{2}(x - 6)$

**57.** $12 + \frac{5}{2}(3 - \frac{2}{3}x) < -4(x - 6)$

Find and graph the solution set to the following nonlinear inequalities.

**58.** $r(r - 3)(6r + 12) \geq 0$

**59.** $w^2 - 1 < \frac{7}{12}w$

**60.** $(4x^2 - 25)(x^2 - 16) \leq 0$

**61.** $(x^2 - 6x + 9)(x^2 - 1) \leq 0$

**62.** $(x - 4)^2(x^2 + 2)^3 > 0$

**63.** $\dfrac{x-1}{x-3} \geq 0$

**64.** $\dfrac{x-1}{x-3} \geq 2$

**65.** $\dfrac{x+3}{x^2 - x - 6} < 0$

**66.** $\dfrac{x-4}{x+5} - 2x \geq 1$

**67.** $\dfrac{3x}{x-1} - \dfrac{2}{x+3} < 1$

**68.** $\dfrac{x-3}{x-4} \geq \dfrac{x+4}{x-6}$

**[2–5]** Solve the following equations and inequalities involving absolute value.

**69.** $\left|\frac{3}{8} - 2x\right| = \frac{3}{4}$

**70.** $|x^2 - 5x| = 50$

**71.** $|x^2 + 1| = 1$

**72.** $\left|\dfrac{x-4}{4}\right| \leq 5$

**73.** $|x^2 - x| = 2$

**74.** $\left|\dfrac{1-2x}{5}\right| < 10$

**75.** $\left|\dfrac{x-2}{4}\right| > 9$

**76.** $\left|\dfrac{2-x}{4}\right| = 9$

**77.** $5 > |-2x - 3|$

**78.** $\left|\dfrac{x+7}{4}\right| \geq \dfrac{2}{3}$

**79.** $\left|3x - \frac{4}{5}\right| > 2$

## Chapter 2 test

Solve the following linear equations by specifying the solution set.

**1.** $7x - 4 = 7(4 - x)$

**2.** $\dfrac{2x-3}{5} = \dfrac{3-4x}{2}$

**3.** $3x - \dfrac{3}{4}x = \dfrac{1}{4}x + 2$

**4.** Find an approximate answer to four digits of accuracy $2.4x - 8.7x = 2(7.2 - 0.2x)$.

Solve the literal equations in problems 5 through 7 for the variable indicated.

**5.** $m = -p(Q - x)$; for $x$

**6.** $P = \dfrac{n}{5}(P_2 - P_1) - c$; for $P_2$

**7.** $\dfrac{x+2y}{3-2y} = x$; for $y$

**8.** A total of \$12,000 is invested, part at 9% and part at 5%. The income from these investments for one year totals \$720. How much was invested at each rate?

**9.** A company has 28 tons of alloy that is 30% copper. How much alloy that is 80% copper must be mixed with this to obtain an alloy that is 50% copper?

**10.** A printing machine can print out 4,000 labels in 20 minutes; a second machine takes 50 minutes to do the same job. How long would it take the printers, running at the same time, to print out a total of 4,000 labels?

**11.** A boat moves at 10 mph in still water. If the boat travels 20 miles downstream in the same time it takes to travel 15 miles upstream, what is the speed of the current?

Solve the following quadratic equations for $x$ by factoring.

**12.** $3x^2 + 4x - 15 = 0$

**13.** $10 + \dfrac{13}{x} = \dfrac{3}{x^2}$

Solve the following equations by extracting the roots.

**14.** $4x^2 - 50 = 0$

**15.** $(3x - 3)^2 = 24$

Solve the following quadratic equations using the quadratic formula.

**16.** $m^2 - 3m - 6 = 0$

**17.** $(z - 1)(z + 1) = \frac{1}{3}(-6z - 7)$

**18.** Use the quadratic formula to help factor the quadratic equation $2x^2 - x - 12$.

**19.** In a right triangle the length of the hypotenuse is 26, and the length of one side is 4 units longer than twice the length of the other side. Find the length of the longer side.

**20.** A manufacturer finds that the total cost $C$ for a product is expressed by $C = 3x^2 - 8$, and the total revenue $R$ by $R = 3x - 2$, where $x$ is the number of units sold. What is the break even point (where total cost = total revenue) to the nearest unit?

**21.** Two printers can be used to print an edition of a newspaper. When both printers run together it takes 12 hours to print the edition. When they are used separately it takes the slower press one hour longer to print the edition than the faster press. Find the time required for the faster press to print the edition alone. Round the answer to the nearest 0.1 hour.

**22.** When flying directly into a 20 mph wind it takes an aircraft one hour longer to travel 400 miles than when there is no wind. Find the speed of the aircraft in no wind.

**23.** Find the domain of the expression $\dfrac{x-1}{x^2-81}$.

Solve the following quadratic equations using substitution.

**24.** $4x^4 - 37x^2 + 9 = 0$ **25.** $x^3 - 7x^{3/2} - 8 = 0$

Find the solution set for each equation.

**26.** $\sqrt{2 + w - w^2} - 1 = w$

**27.** $\sqrt{5n + 5} - \sqrt{13 - n} = 2$

**28.** Solve the formula for $b$: $r = \sqrt{\dfrac{A}{\pi} - Ab}$.

Find and graph the solution sets to the following linear inequalities.

**29.** $4x - 3(2x - 3) < x$ **30.** $x - 5 \le 2x - \frac{1}{2}(6 - x)$

Find and graph the solution set to the following nonlinear inequalities.

**31.** $(x - 3)(x^2 - 4) > 0$

**32.** $(x^2 - 4x + 4)(x^2 - 1) \le 0$

**33.** $\dfrac{x - 10}{x - 3} > 2$ **34.** $\dfrac{x + 3}{x^2 - 2x - 8} \ge 0$

Solve the following equations and inequalities involving absolute value.

**35.** $|5 - 2x| = 8$ **36.** $|3x - 2| \le 10$

**37.** $\left|\dfrac{2 - 3x}{4}\right| > 2$ **38.** $|5x - 1| \ge 6$

**39.** $3 > |2x - 3|$ **40.** $|x^2 + 2x| = 10$

Solve the following equations.

**41.** $\dfrac{2x - 3}{3} + \dfrac{5 - x}{4} = 1 - \dfrac{2x + 3}{6}$

**42.** $\dfrac{2x}{x + 1} + \dfrac{2(x + 2)}{x} = 3$

**43.** $\dfrac{2}{x + 1} + \dfrac{2(x + 2)}{x} = 3$ **44.** $\dfrac{1}{x + 1} + \dfrac{2}{x + 2} = 1$

**45.** $\dfrac{5}{2x - 3} = \dfrac{4}{x + 1}$ **46.** $\dfrac{5}{2x - 3} = \dfrac{4x}{x + 1}$

**47.** $\dfrac{2x - 5}{x + 2} = 3$ **48.** $\dfrac{2x - 5}{x + 2} = 3x$

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